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Phenomena at the reflection of ultrasonic waves by solid material. 1927.

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PHENOMENA AT THE REFLECTION OF ULTRASONIC WAVES BY SOLID MATERIALS

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by D.K. Froman, B.Sc.

A Research conducted under the
supervision of Dr. R. W. Boyle.

Presented to the Committee on Graduate Studies of the University of Alberta in partial requirement for the Degree of Master of Science.

April, 1927.

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INTRODUCTION.

The object of the investigation reported here was to study certain types of phenomena which may be exhibited at the reflection of ultrasonic waves incident in liquid media on solid reflectors.

It has been convenient to divide the investigation into two main parts, viz;

Part I: A study of the cavitation which may take place in stationary waves produced by reflection, this being a continuation of the work commenced by Boyle and Taylor; and

Part II: A study of the way in which the amount of the reflected energy is dependent upon the thickness of the reflecting body.

Incidentally, it was possible to observe certain relations existing between the overtones and fundamental frequency of an ultrasonic transmitter when generating at its resonant frequencies; to make a very limited survey of the pressure of radiation in stationary waves produced by reflection; and to study the piezo-electric properties of some quartz mosaics.

The Apparatus of Experiment.

In these experiments the ultrasonic beams have been produced by the piezo-electric action of quartz discs, excited by the electrical generating circuit shown diagramatically in

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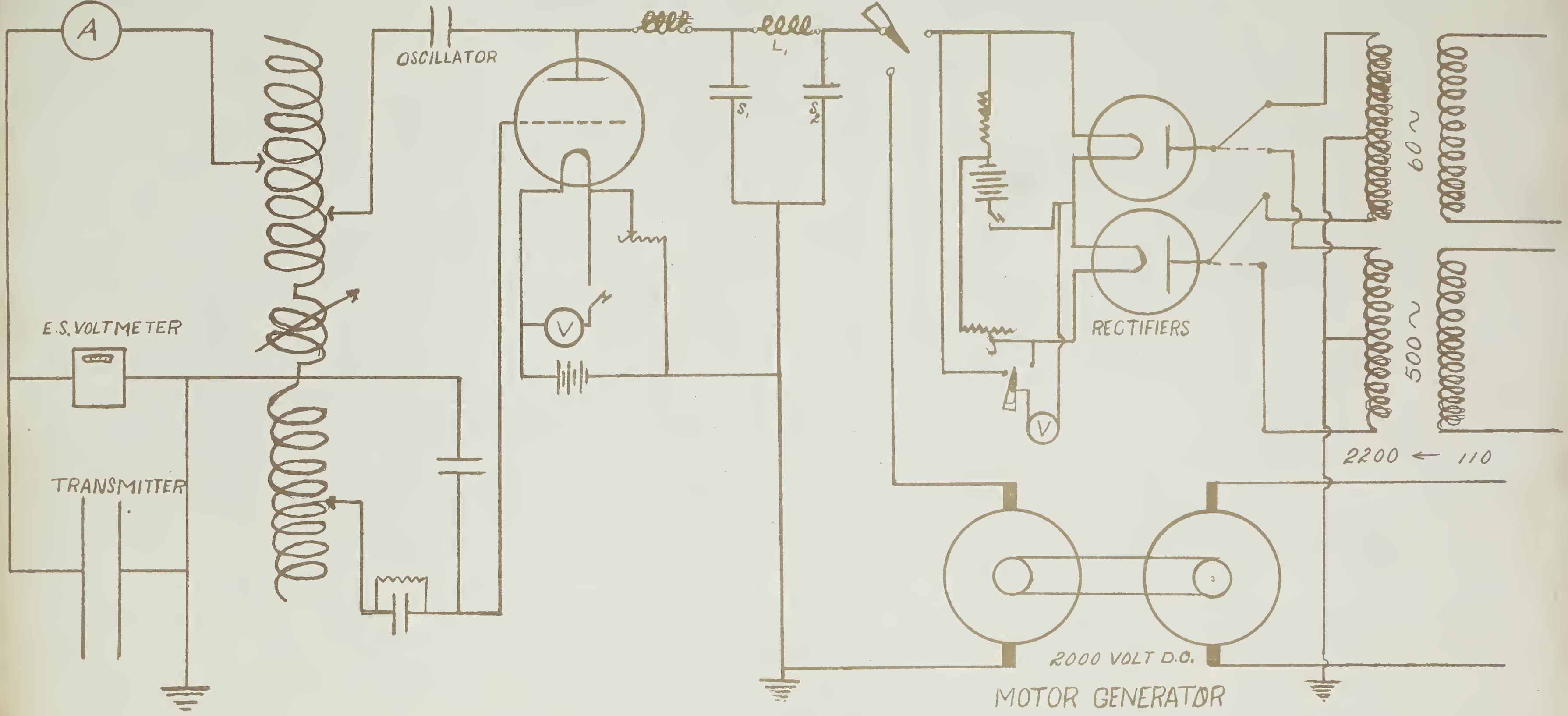
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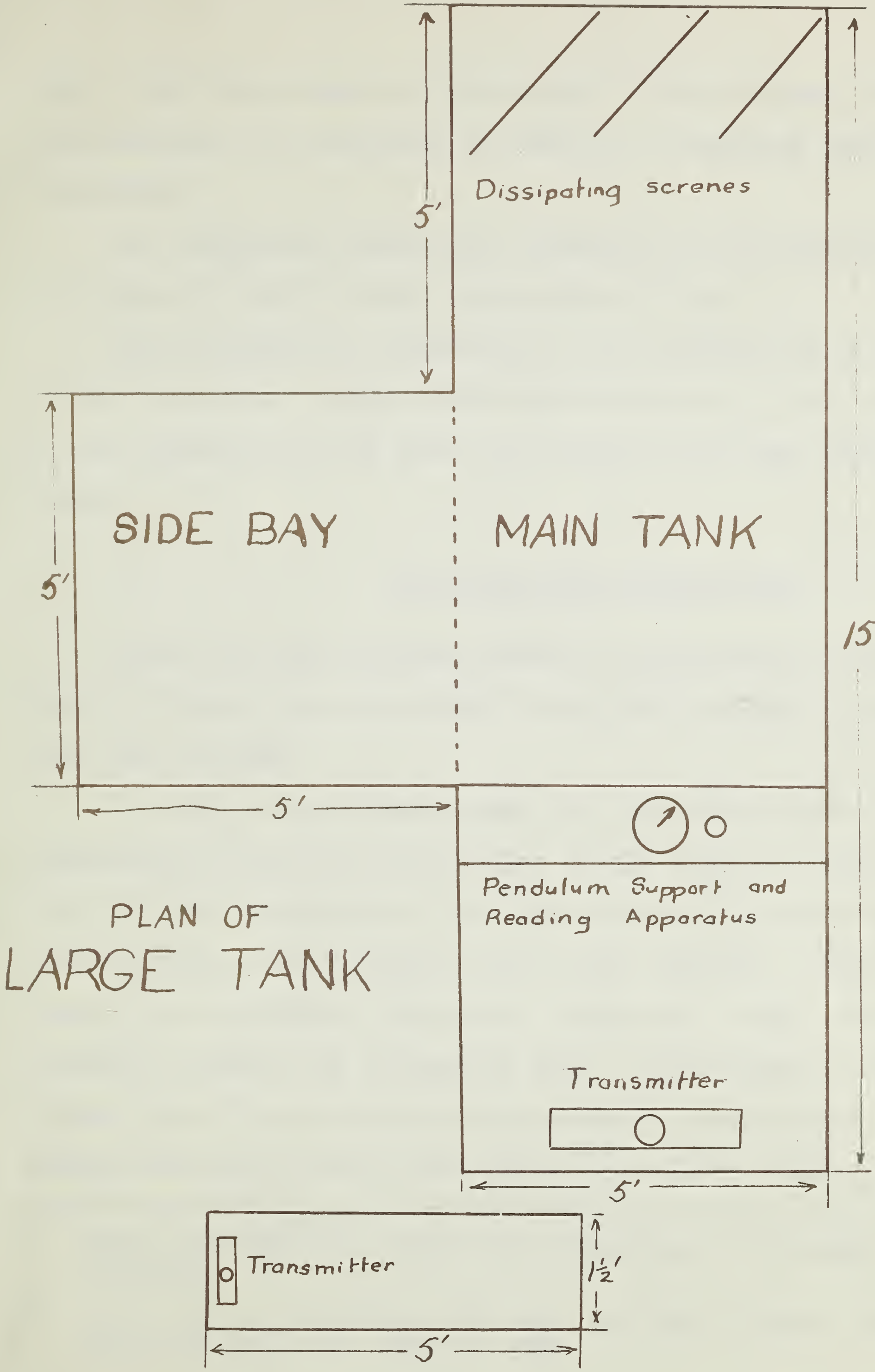
MOTOR GENERATOR

FIG I



PLAN OF SMALL TANK

F g 2

fig. 1 and fully described elsewhere[1]. The frequency of the oscillations was determined by means of a Precision Hertzian Wave-meter.

The ultrasonic waves were propagated in liquids contained in either of the two tanks represented in fig. 2.

When required the intensity of the radiation at any point in the ultrasonic energy-field was determined by the pressure of the radiation on the vane of a torsion pendulum, described later.

The Ultrasonic Transmitter.

Since the piezo-electric method of generating ultrasonic waves has often been described[2] it will be referred to here only very briefly.

As used in these experiments, the transmitter consisted essentially of one or two circular quartz discs, or mosaics, the plates of which were of uniform thickness cut with faces perpendicular to an electric axis of the crystal. These discs were piled alternately with steel plates in a "pile of plates" condenser enclosed in a metallic case. The number of steel plates used is one more than the number of quartz discs. The casing was made of cast iron with an open front, except for a

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[1] Boyle and Lehmann: Report on Ultrasonics to Canadian Research Council.

[2] Loc. cit. 1, and Boyle, Lehmann, and Reid, Trans. Roy. Soc. Can. 3rd Ser. XIX, 1925, p. 166.

small rim about the edge upon which the front steel plate of the condenser could rest. This steel plate formed the radiating face of the transmitter. In constructing a transmitter the back of the casing, which is bolted on, was removed and the front steel plate placed in position, the casing being face down. The case and front plate were then heated and a small amount of a molten insulating material, a mixture of paraffin and resin in the proportion of 2 to 1, was poured over the front plate. A quartz disc was then placed on the steel plate and forced tightly against it leaving only a very thin layer of the insulating material between ~~them~~ the quartz and steel. Another small amount of the molten insulator was poured over the quartz and the second steel plate pressed down upon it. This was continued until all of the plates were in position. Alternate steel plates were then connected together by ~~electrical conductors~~ wires. Enough insulating material to cover the back steel plate was then poured into the casing and a heavy weight placed upon the back plate. The pressure squeezed out any excess of the mixture so that only a thin layer was left between the discs. As the transmitter cooled, the mixture hardened, holding the contents in position and cementing all the discs together. When cool the weights were removed, the case completely filled with the mixture, allowed to cool, and the back bolted on. Thus the ultrasonic generator is really a parallel plate condenser with quartz as dielectric. Care was taken to insert the quartz discs so that each had its positive face against steel plates of like polarity, so that

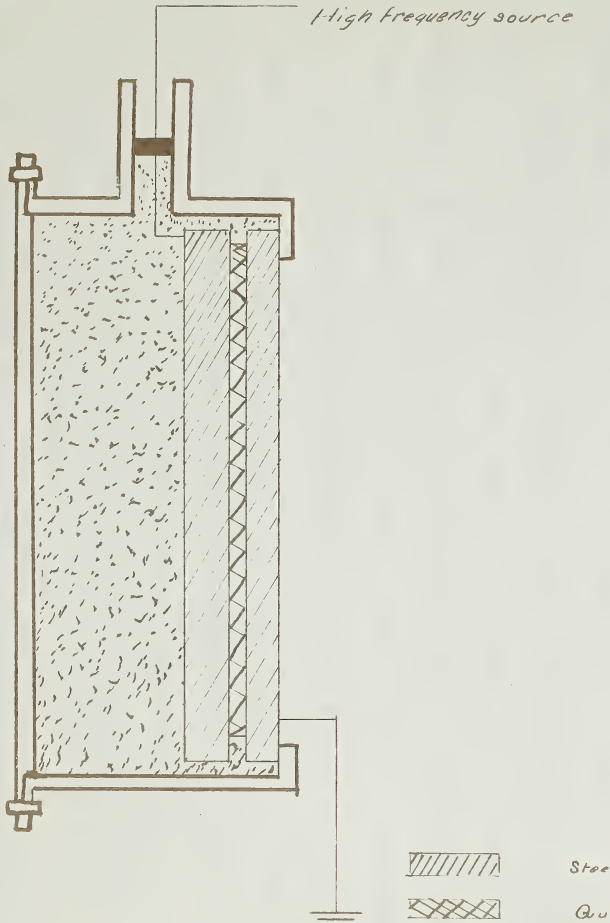


Fig. 3a

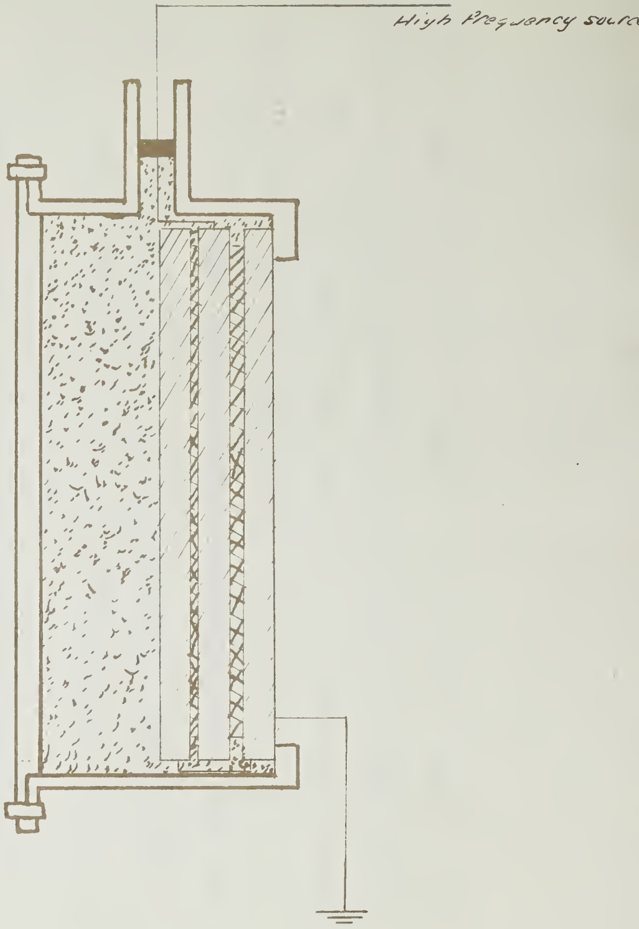


Fig. 3b

▨ Steel plates

▩ Quartz disc

░ Insulating material.

▭ Cast iron casing

when a potential was applied all of the discs would either dilate or contract together and none act in opposition.

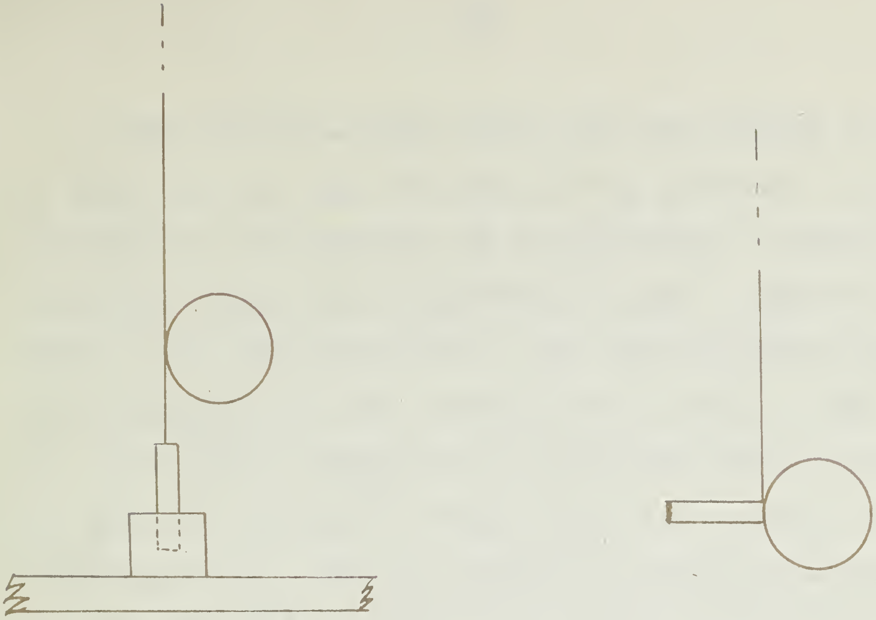
The steel plates were connected to the electrical oscillating circuit as is shown in fig. 1.

Figs. 3a and 3b show cross sections of a "double-steel-plate" transmitter and a "pile-of-plates" transmitter, the latter containing two quartz and three steel plates, respectively.

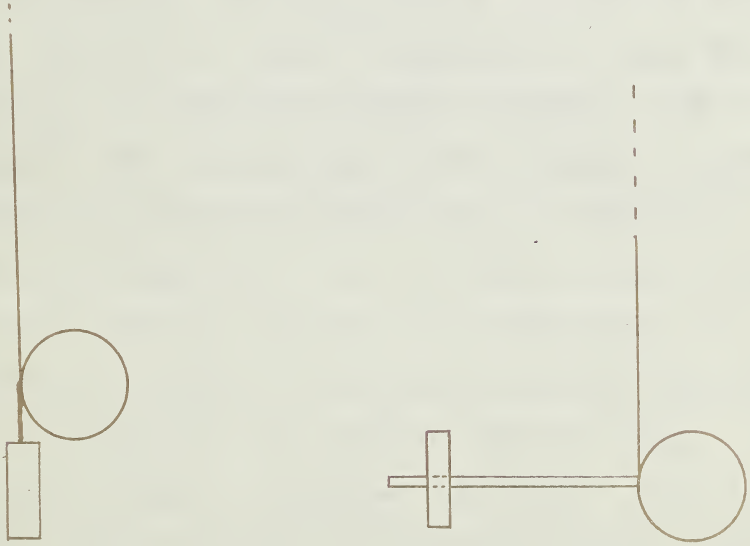
If an electric difference of potential be applied to opposite faces of a plate of quartz, cut perpendicular to an electric axis of the crystal, the plate will expand or contract in the direction of the electric field by an amount directly proportional to the potential difference applied.

By connecting the transmitter to an electric oscillating circuit it is possible to apply an alternating potential difference to the faces of the quartz disc, thus causing it to execute longitudinal vibrations with the same frequency as that of the electric oscillations. The faces of the quartz mosaics were cemented under pressure to the steel plates, and therefore, as the quartz is made to vibrate it carried the steel plates into vibration with it. Hence the front steel plate becomes a source of compressional waves in the medium with which it is in contact.

The steel plates employed in the transmitters were circular; therefore the emission of ultrasonic waves by the front plate is somewhat analogous to the emission of plane waves of light from or through a circular aperture; and the mathe-



SINGLE VANE DOUBLE VANE



SINKER TYPE COUNTER-BALANCED TYPE

TYPES OF PENDULA

Fig. 4

Thus, if the energy density of the incident beam be E ergs per c.c., and the ratio of the reflected to incident energy be R, the pressure in the direction of travel of the incident energy upon a partition at right angles to the beam will be (E + RE) dynes/cm.^{2}. Let the absorption coefficient of the material of the partition be α, that is the intensity I at a point a distance t from the face of the pendulum is given by $I = I_0 e^{-\alpha t}$, where I_0 is the intensity of the entering waves. Then the amount of energy transmitted will be (E - RE) $e^{-\alpha t}$ ergs/c.c., where t is the thickness of the Pendulum, and due to it the back pressure upon the partition, in the direction against the incident energy, will be (E - RE) $e^{-\alpha t}$ dynes/cm.^{2}. Hence the resultant pressure in the incident direction becomes E + RE - (E - RE) $e^{-\alpha t}$ dynes/cm.^{2}. If the partition is thin enough to neglect absorption this expression reduces to 2 RE dynes/cm.^{2}. If reflection is perfect, R = 1, and the pressure becomes 2 E dynes/cm.^{2}; whereas, if there is no reflection, (R = 0), and absorption is perfect, the pressure becomes E dynes/cm.^{2}; i.e. the pressure of the incident radiation only.

Several types of pendulums were used. These are shown in fig. 4 under the names by which they will be designated in this report. The mounting and manipulation of pendulums has been described fully elsewhere[3] and therefore will be only briefly ~~described~~ refered to here.

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3. Rayleigh, Phil. Mag. (6), 3, p 338, 1902.

The pendulums were suspended by a phosphor bronze wire or strip, attached to a torsion head provided with pointer and scale. The vertical vane of the pendulum was set perpendicular to the axis of the ultrasonic beam when the transmitter was not acting. The transmitter was then excited causing a deflection of the pendulum, and the torsion head was twisted until the pendulum assumed the same position as before. This amount of twist will be known herein as the "pendulum reading".

It has been shown[6] that, in all practical cases, the pressure at the point of suspension, $p = c\theta$ .... (1) very approximately, where c is a constant and θ is the pendulum reading. Moreover it was shown that energy density at the point of suspension, E , is given by

~~$E = K\theta$ approximately .... (2)~~

~~where K is a constant~~.

$E = R\theta$ approximately .... (2a)

where R is a constant.

Also $$\Phi = \frac{4\pi I}{T^2} \qquad \text{.........................} \quad (2b)$$

where Φ = the torsional constant of the suspension, I the moment of inertia of the pendulum and T its period of oscillation in air.

In the case of the double-vane pendulum $R = \dfrac{\Phi\,\theta}{2K(\pi a_v^3 - 2ta_h^2)}$ where a_v and a_h are the radii of the vertical and horizontal vanes respectively and t is the thickness of the horizontal vane. K is an empirical constant which may be determined from a curve by Boyle and Lehmann.

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[6] Boyle and Lehmann, Report on Ultrasonics to Canadian Research Council, 1922.

Part I: Cavitation.

On theoretical considerations[7], if, at any frequency, the amplitude of the ultrasonic wave be great enough, breaks or discontinuities might be expected to occur in the medium carrying the wave train; also it might be possible that the liquid in contact with the vibrating face of the transmitter would be unable to follow the motions of the face and a "cave" or partial vacuum be formed.

Let p_0 be the hydrostatic pressure of the medium at a point in the path of the ultrasonic beam, and p' the vapour pressure of the liquid at the existing temperature. At any given point in the beam there are successive condensations and rarefactions which alter the resultant pressure at that point. Let p be the maximum harmonic pressure due to the wave motion at the point under consideration. Then, assuming the existence of a bubble however small, the resultant pressure at that point ranges between a maximum value of $(p_0 - p') + p$ and a minimum value of $(p_0 - p') - p$.

When $p \geq p_0 - p'$ there will be an instant or interval of time during each wave period at which the resultant pressure is negative equal to zero, and the medium is in tension. The vapour pressure of the liquid, under this condition, might cause a break in the liquid and a bubble to be formed provided there ~~be~~ were time enough for such a formation to take place. Cavitation

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[7] Boyle - Trans.Roy.Soc.Can. 3rd Ser. XVI, 1922, pp.157-161.

of this particular type would set in therefore when $p = p_0 - p'$.

The amplitude of pressure in a longitudinal wave motion is given by[8]

$$p = 2\pi a n v \rho ,$$

where a is the amplitude of displacement of the vibration, n the frequency, v the velocity of propagation, and the density of the medium.

Suppose the sound wave to be of the form

$y = a \sin \frac{2\pi}{T} (t - \frac{x}{V})$ where y is the instantaneous displacement, a the amplitude, T the period, V the velocity, x the direction and distance of advance, and t the time.

The kinetic energy per unit Volume, T, is given by

$$T = \tfrac{1}{2}\rho \left(\frac{dy}{dt}\right)^2$$

$$= \tfrac{1}{2}\rho \left(\frac{2\pi}{T}\right)^2 a^2 \cos^2 \frac{2\pi}{T} (t - \frac{x}{V}).$$

Since the sum of the kinetic and potential energies is constant, the total energy per unit volume will be the maximum kinetic energy $= \frac{1}{2}\rho \left(\frac{2\pi}{T}\right)^2 a^2$.

The power trahsmitted across unit area at right angles to the direction of transmission is

$$W = \tfrac{1}{2}\rho \left(\frac{2\pi}{T}\right)^2 a^2 V = \tfrac{1}{2}\rho (2\pi an)^2 V.$$

But $p = 2\pi anv\rho$,

$$\therefore W = \tfrac{1}{2}\frac{p^2}{\rho v} .$$

If $p = p_0 - p'$ the medium would be in tension and discontinuities might appear and be evidenced by the formation of

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[8] "Dynamical Theory of Sound", Lamb.

bubbles. Transmission would then become discontinuous. Hence the approximate upper limit of the power which could be transmitted is given by

$$W = \frac{\frac{1}{2}(p_0 - p')}{\rho v} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (3)$$

and when this power is being transmitted bubbling should take place.

The problem attempted here was to determine the actual power required to produce bubbling in a liquid and to compare it with the amount calculated as above; and also to study the effect upon the phenomenon of the stationary waves produced by reflection.

The method was similar to that followed by Boyle and Taylor[9] who found that bubbling occurred at a much lower energy emission than is required by the simple theory of equation (3).

Using as the liquid medium to propagate the ultrasonic waves a commercial lighting naptha, experiments were performed to determine the minimum high frequency voltage applied to the transmitter, and hence the lowest energy emission, at which bubbling of the liquid would take place; and also to study the effect on this phenomenon of stationary waves produced by reflection of the incident beam.

A double-steel-plate transmitter, known hereinafter as transmitter No. 2 was placed in the small tank, fig. 2, which was nearly filled with naptha.

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[9] Physical Rev. 2, 27, April, 1926, p. 518.

The transmitter was set to oscillate at one of its resonant frequencies, viz. 44,600 cycles per second. The high frequency voltage applied to the transmitter was such that bubbling was produced copiously at the beginning of each experiment. The transmitter voltage was then gradually reduced until bubbling was just evidenced. This was carried out under different sets of conditions in order to determine the best conditions for this type of "cavitation" to take place. Trials were made with the transmitter face both vertical and horizontal. The effects of agitating the liquid and the effects of a solid reflector placed in the beam were studied.

When the transmitter was set with its face vertical, so that the beam was directed horizontally down the longitudinal axis of the tank, bubbles were formed most copiously on or near the axis of the beam close to the transmitter. The bubbles were driven away from the transmitter by the action of radiation pressure, and eventually rose to the surface some distance away. The maximum distance from the transmitter at which bubbles were formed could not be accurately determined because of this movement of the bubbles along the beam, which movement near the axis was quite rapid; but it was evident that the higher the transmitter voltage the greater was the maximum distance at which the formation of bubbles could occur. Trials were made to determine the lowest transmitter voltage which would produce bubbling both with the fluid at rest and under agitation. In accordance with the experiments of Boyle and Taylor, more copious bubbling

could be obtained when the fluid was agitated, and a lower value of the minimum voltage required to cause bubbling was obtained if the liquid were stirred during the experiment. This effect of agitation was found to be the case under all other conditions which ~~were~~ could be imposed.

When a solid reflector was placed in the path of the beam, the bubbles collected at the nodes of the stationary waves and rose to the surface in curtains parallel to the face of the reflector. To be certain that these curtains were formed one-half wave length apart, and at the nodes, a stationary wave dust figure experiment[10] was performed to obtain a picture of the energy field. The bubbling being in progress, fine cinder dust was sifted into the energy field. The dust particles were driven to the vertical nodal planes along which they sank in curtains, forming lines parallel to the face of the reflector and a half-wave length apart upon the bottom of the tank. Through a glass window in one side of the tank it was observed that curtains of both cinders and bubbles were formed in the same positions.

On facing the transmitter upwards so that it radiated towards the surface of the naptha, stationary waves were formed by downward reflection from the air-naptha surface. The bubbles collected in approximately horizontal layers parallel to the surface a half wave length apart. They did

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10. Boyle, Lehmann and Reid, Transactions Royal Society of Canada, Vol. XIX, Sec. III, 1925.

not rise readily, but tended to remain suspended in these layers, except when very near the transmitter where the radiation pressure forced them to move upwards. Any particular bubble, if watched, was observed to grow larger until its buoyancy became so great as to cause it to rise, sometimes clear to the surface sometimes to a layer further up. Some bubbles were observed to unite together in the layers, others to grow larger whether they were seen to unite with others or not. This action may have been due to one or both of two causes, viz: the bubble under observation united with smaller invisible bubbles, or the evaporation of the naptha fluid into the bubble took place.

If the transmitter were stopped oscillating the bubbles suspended in layers in the liquid immediately rose to the surface. According to ~~Rayleigh's~~ the theory of the pressure of radiation the energy emitted by the transmitter would tend to cause the bubbles to move to the surface, whereas that reflected from the surface would tend to cause them to move downwards. But the reflection was not perfect ~~and~~ a certain amount of the energy ~~was~~ being lost in the reflection and in traversing the distance from the layer to the surface and back again. Hence the resultant radiation pressure upon the bubble would ~~seem to~~ be toward the surface. This pressure, combined with the buoyancy of the bubbles, should

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~~11. Lord Rayleigh, Phil. Mag. (6), 3, p. 338, 1902.~~

cause the bubbles to rise more rapidly toward the surface while the transmitter was in oscillation than when it was inactive, whereas it was found that the bubbles would hardly rise at all when the transmitter was oscillating. A possible explanation might be drawn from Addendum III although the work there mentioned needs to be confirmed. The brief survey of the pressure of radiation in stationary waves[12], as described in Addendum III, seemed to indicate, however, that the pressure is not the algebraic sum of the incident and reflected pressures, but that it varies as some harmonic function of the distance from the reflector in such a way that its value is zero at both nodes and antinodes, and is in opposite directions on each side of either a node or an antinode. At the side of a node nearer the reflector the pressure was found to be away from the reflector and increasing in value to a point midway between the node and antinode. Presumably, therefore, the bubbles rose above each node until the pressure of radiation away from the surface was equal to the force of buoyancy and there formed a layer.

Experiments were carried out at different frequencies. With the transmitter voltages imposed, it was only when the transmitter was in mechanical resonance that it was possible to obtain bubbling. In fact the resonant frequencies of the transmitter could be determined by varying the frequency of oscillation slowly and observing the particular frequency at which bubbling was most abundant. The frequencies of

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[12] Addendum (2), ibid.

resonance determined in this way agreed with those determined by means of a torsion pendulum.

Out of a large number of experiments performed the following may be quoted as a typical example.

Operating the transmitter at a frequency of 171400 cycles/sec. bubbling was obtained in the naptha by the use of a reflector at a high frequency transmitter voltage of 3400 volts. Cavitation was copious, the bubbles being driven by the pressure of radiation, six or seven inches from the face of the transmitter. Good stationary waves, as evidenced by curtains of bubbles were formed if the reflector were held anywhere from six to twelve inches from the transmitter. The reflector was removed and the voltage gradually decreased, the naptha ~~was~~ being frequently agitated. The ~~lowest~~ minimum high frequency voltage at which bubbling was observed was 1250 volts. The reflector was replaced and the voltage raised to 3400 volts and then decreased. This was repeated for several positions of the reflector. When it was near the radiating face, about 1 cm. distant, bubbling could be obtained at a lower voltage than when it was more distant. The lowest voltage at which bubbling occurred was 1150 volts. The temperature of the naptha was 19.9° C.

At a frequency of 42800 cycles per second the minimum transmitter voltage at which bubbles were obtained in the stationary waves after agitating the liquid was 1500 volts.

At a frequency of 172500 cycles per second the minimum transmitter voltage, ~~was~~ under similar conditions, was 900 volts.

It was noticed that the bubbles were smaller at the higher frequencies. In each of the above cases the axis of the beam was horizontal and a glass-plate reflector was used to produce the stationary waves.

A sample of the gas in the bubbles was obtained in order to determine its constitution. The transmitter was set with its face upwards and just beneath the mouth of a funnel, through which the bubbles could rise and be collected by displacement. The funnel cut off the action of the reflection from the liquid surface to a great extent, and, with the aid of occasional agitation, the bubbles rose copiously into a glass cylinder previously filled with the naptha and inverted over the neck of the funnel. Approximately 200 c.c. of the gas were collected, and, judging from the rate at which bubbles formed and rose, and from their size, one would estimate that the 200 c.c. of gas would be collected in about an hour at the most. But it required more than fifteen hours running to collect this much gas. It was clear that much of the contents of the gas was absorbed into the naptha as the bubbles rose through the naptha column. So it was desired to analyse the gas resulting from Cavitation.

A sample of air was first analysed. The volume of air admitted to the apparatus = 49.15 c.c.

After contact with pyrogallic acid, which removed the oxygen, 39.25 c.c. remained.

Hence

$$\text{oxygen present in air} = \frac{49.15 - 39.25}{49.15} \times 100 = 20.0\%$$

$$\text{and nitrogen present in air} = \frac{39.25}{49.15} \times 100 = 80.0\%$$

The nitrogen was expelled and a sample of the gas to be analysed was admitted. Volume = 39.87 c.c.

After passage through pyrogallic acid the volume remaining = 33.05 c.c. Hence the oxygen present = $\frac{39.87 - 33.05}{39.87} \times 100 = 17.1\%$

Then a sample of 80.00 c.c. of air was admitted in order to burn any organic vapors present in the gas.

Volume of specimen and air after burning = 108.5 c.c.

After contact with potassium hydroxide vol. = 105.5 c.c.

$\therefore$ There were 3.0 c.c. of carbon dioxide present. The mixture was passed through pyrogallic acid to remove the excess of oxygen. The volume remaining = 92.55 c.c. This is the amount of nitrogen remaining. The nitrogen admitted with the air = 80.0% of 80.00 c.c. = 64.00 c.c.

$\therefore$ the amount of nitrogen in the specimen = 92.55 - 64.00 = 28.55 c.c.
i.e. $\frac{28.55}{39.86} \times 100 = 71.6\%$.

$\therefore$ the amount of organic vapours present = $100\% - 71.6\% - 17.1\%$
$= 11.3\%$.

The results may be summarized as follows:

Amount of oxygen present -	17.1% by volume,
Amount of nitrogen present -	71.6% " "
Amount of organic vapour present -	11.3% " " .

It will be noticed that the oxygen and nitrogen were present in the ratio 17.1 : 71.6 , i.e. 1 : 4 approximately, which is near the oxygen-nitrogen ratio for air. Hence about 89% of the gas collected was air. At the temperature at which the experiment was carried out, about 15° C, the vapour pressure of the naptha was sufficient to account for the presence of 11.3% of organic vapours.

In order to make determinations of energy emissions there were some special circumstances which had to be considered. It would be impossible to make satisfactory measurements in the small tank in which the cavitation experiments were carried out since reflections from the end, sides and bottom of the tank, and from the surface of the liquid, would completely distort the energy field. Hence measurements had to be made in the large tank already referred to, the transmitter being operated under the same conditions as when bubbling took place in the naptha. The large tank was filled with water as it was obviously too costly to fill it with naptha.

A horizontal cross-sectional energy survey of the beam was taken with a single-vane air pendulum at a frequency of 172,500 cycles per second, this type of pendulum being used because with it measurements could be taken speedily and conveniently. But in order to make an absolute determination of energy emission this survey needed to be known in terms of the deflections of a double-vane pendulum, since with the double vane device the moment of inertia and the torsional constant of the suspension can most readily be computed. The deflections of the double and single-vane pendulums are directly proportional to one another so that one reading with each, taken at the same position in the energy field and under the same conditions, was sufficient to make the single vane readings reducable to the double vane.

The vertical cross-sectional energy survey of the ultrasonic beam is always very similar to the horizontal survey and can be assumed identical. The horizontal section was taken

across the middle of the beam.

A double-vane lead pendulum was used, the dimensions of which were:-

diameter of each vane = 2.51 cms.,
thickness " " " = 0.164 cms.,
mass of pendulum = 17.9 grams.

The pendulum was first set in the centre of the beam vertically, and then moving the pendulum to different horizontal positions the energy distribution in a horizontal plane in the cross-section 14.5 cms. from the transmitter was taken. This distance was chosen because it was a little greater than the maximum distance from the transmitter at which the furthest interference maximum of intensity could occur on the axis. A calculation showed that at 14.5 cms. from the transmitter energy measurements of the main beam could safely be taken.

The voltage applied to the transmitter was 1400. The readings taken are given in Table I, and the pendulum readings are plotted against horizontal positions in Fig. 5.

To make an absolute determination of the energy emission it was necessary to know the torsional constant of the suspension, for which the moment of inertia of the pendulum must be known. For this purpose the period of oscillation of the pendulum was taken in air, and again with a brass cylinder attached centrally to the pendulum by a short, stiff, wire in line with the axis of the cylinder.

Let I_1 = the moment of inertia of the cylinder,
I = " " " " " " pendulum,
T = the period of the pendulum alone,
T_1 = " " " " " and cylinder.

Then $I = \frac{I_1 \; T^2}{T_1^2 - T^2}$, and since by equation (2b),

$$\phi = \frac{4\pi^2 I}{T^2}, \therefore \phi = \frac{4\pi^2 I_1}{T_1^2 - T^2}$$

The mass of the cylinder, m, = 43.6 grams.

The radius of the cylinder, r = 2.01 cms.

Hence the moment of inertia of the cylinder = $\frac{mr^2}{2}$

$$= \frac{43.6 \times (2.01)^2}{2} \text{ gram cms.}^2.$$

In determinations of the period of the pendulum alone 30 oscillations occurred in 403 secs. Hence the period, $T, = \frac{403}{30} = 13.4$ secs. With the pendulum and cylinder together 20 oscillations occurred in 512 secs. Hence the period

$$T_1, = \frac{512}{20} = 25.6 \text{ secs.}$$

Hence $= \frac{4\pi^2 \times 43.60 \times (2.01)^2}{2 \; (25.6)^2 - (13.4)^2} = 7.33$ ergs, and from equation (2a) p. 7, the maximum energy density corresponding to the maximum pendulum reading of 1.85 radians, is

$$E = \frac{1}{2 \times 0.95} \quad \frac{7.33 \times 1.85}{\pi(1.255)^3 - 2 \times 0.164 \times (1.255)^2} = 1.25 \text{ ergs/c.c.}$$

K is determined as 0.95 from the empirical curve of Boyle and Lehmann previously referred to.

The average energy emitted per square centimeter of the face of the transmitter can be estimated by means of the energy survey curve and equation (2); since the total energy in a section of the medium one cm. in thickness at right angles to the beam is given by

$$\int_0^{E\ max.} \overline{W}\ r^2\ dE\ .$$

Here W max. is the maximum energy density in the beam, and r is the distance from the axis of the beam to the position of the element considered.

By means of the curve of pendulum reading against horizontal position and equation (2), values corresponding to given values of E may be determined and the integral evaluated by means of Simpson's Rule.

Thus the total energy emitted per second from the transmitter can be computed and an approximate estimate can be made of the energy emission per unit area of that point on the face of the transmitter having the maximum emission.

$\int_0^{E\ max.} \overline{W} r^2\ dE$ may be evaluated by Simpson's Rule.

Let the curve of energy density across the beam be divided into ten equal parts by eleven ordinates. The values of the energy, E, at each ordinate and the corresponding values of r and $\overline{W}r^2$ are given in Table II.

By Simpson's Rule the integral is the product of one-third of the interval between the ordinates and the sum of the first and last ordinates, plus twice the sum of all of the other odd ordinates, plus four times the sum of all of the even ordinates;

i.e. $\int_0^{E_{max.}} \overline{\pi} r^2 \, dE = \frac{1}{3}.\frac{Emax}{10}.\Big[\left(\text{ordinate 1 + ordinate 11}\right) + 2 \left(\text{sum of other odd ordinates}\right) + 4 \left(\text{sum of all even ordinates}\right)\Big]$

where 10 is the even number of equal parts into which E max is to be divided.

TABLE I.

Horizontal Position, x, in Cms.	Pendulum Reading in degrees	Pendulum Reading θ, in radians
28.0	0.0	0.000
29.0	12.9	0.225
30.0	15.9	0.278
31.0	32.8	0.572
32.0	53.6	0.936
32.5	80.6	1.407
33.0	92.5	1.601
33.5	105.8	1.846
34.0	98.0	1.710
34.5	67.5	1.178
35.0	64.0	1.126
36.0	30.0	0.523
37.0	16.0	0.279
38.0	9.0	0.157
39.0	4.0	0.0698
40.0	1.0	0.0175

Fig: 5

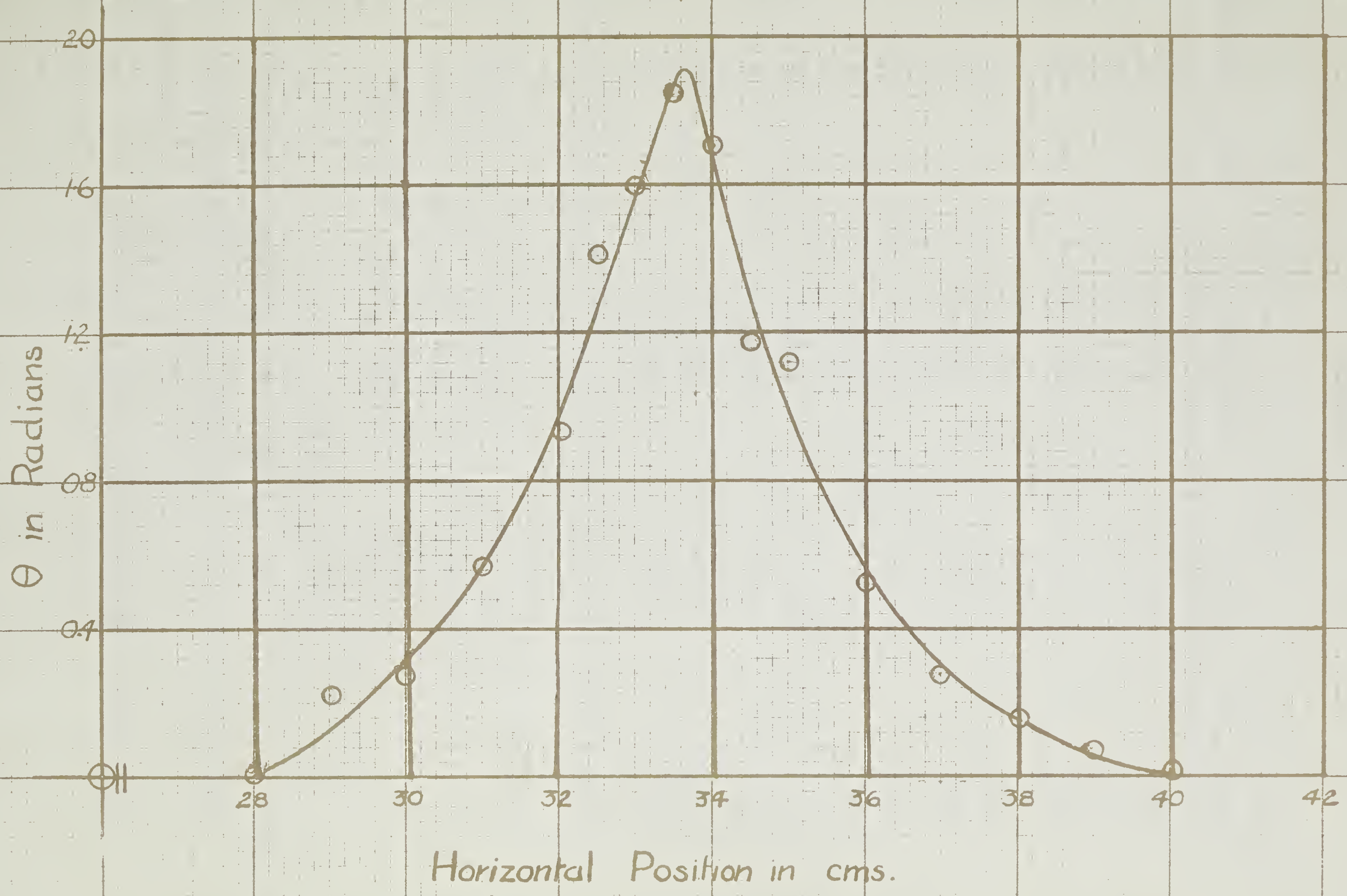
20
16
12
08
0.4
Θ in Radians
28
30
32
34
36
38
40
42
Horizontal Position in cms.

TABLE II.

Ordinate Number	E in ergs/c.c.	r in cms.	πr^2 in cms.^{2}
1	1.250	0.00	0.00
2	1.125	0.55	0.95
3	1.000	0.80	2.05
4	0.875	1.25	4.91
5	0.750	1.30	5.31
6	0.625	1.60	8.05
7	0.500	1.90	11.35
8	0.375	2.35	17.35
9	0.250	3.00	28.25
10	0.125	4.25	57.70
11	0.000	6.80	145.20

Ordinate No. 1 + ordinate 11 = 145.20

2 (sum of other odd ordinates) = 93.92

4 (sum of all even ordinates) = 351.84

Total = 590.96.

.·. the energy contained in a section one cm. thick and perpendicular to the beam, at a distance of 14.5 cms. from the transmitter is

$$\int_0^{E_{max.}} \pi r^2 \, dE = \frac{1}{3} \times 0.125 \times 590.96 = 25.6 \text{ ergs.}$$

The diameter of the face of the transmitter was 10.2 cms. and hence the area of the face was $\pi(\frac{10.2}{2})^2$ = 80 cms.^{2} approximately.

Therefore the average energy emitted per square cm. of the transmitter face = $\frac{25.6}{80}$ = 0.32 ergs.

The power in watts/cm.^{2} is $\frac{EV}{10^7}$, where V is the velocity of propagation of the energy, which in water is 1.48×10^5 cms./sec.

Hence the average power emitted by the face was

$$\frac{0.32 \times 1.48 \times 10^5}{10^7} = 4.74 \times 10^{-3} \text{ watts/cm.}^2$$

Since the velocity of sound in naptha is nearly equal to that in water the power is about the same in either medium.

This energy measurement was made at 1400 volts applied to the transmitter. Since the energy emission varies as the square of the voltage[14], the average power emitted by the transmitter, when bubbling occurred in the naptha at a voltage of 900 volts, was

$$(\frac{900}{1400})^2 \times 4.74 \times 10^{-3} = 2.8 \times 10^{-3} \text{ watts/cm.}^2.$$

The maximum power at 14.5 cms. from the transmitter operating at 1400 volts was

$$\frac{1.25 \times 1.48 \times 10^5}{10^7} = 19.2 \times 10^{-3} \text{ watts/cm.}^2,$$

1.25 ergs/c.c. being the maximum value of the energy density in the beam. Hence at 900 volts applied to the transmitter the maximum power was $(\frac{900}{1400})^2 \times 19.2 \times 10 = 8.0 \times 10^{-3}$ watts/cm.^{2}.

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[14] Boyle and Lehmann, Report to Canadian Research Council, 1922.

In these experiments the reflector was placed about fifteen cms. from the transmitter and bubbling occurred over most of the region between the reflector and the transmitter. If there were perfect reflection, which of course was not the case, the energy at an antinode would be twice as great as it would be if there were no reflection. Hence, assuming perfect reflection the power which in these experiments was able to produce cavitation was $2 \times 8.0 \times 10^{-3}$ watts/cm.^{2} = 16.0×10^{-3} watts/cm.^{2}. This figure is in very good agreement with the previous tentative result of Boyle and Taylor[15] who obtained a minimum value of the power required to produce bubbling in naptha of 18×10^{-3} watts/cm.^{2}. They did not use a reflector in the experiments in which they took quantitative measurements.

The power required, by simple theory, to produce cavitation is given by equation (3):

$$W = \frac{\frac{1}{2}(p_0 - p')^2}{\rho V} .$$

p , the hydrostatic pressure was approximately 70 cms. Hg.

p', the vapour pressure of the naptha at the temperature of experiment was measured and found to be 10 cms. Hg.

ρ , the density of the naptha was 0.70 grams/c.c., and

V , the velocity of sound in naptha, was 1.40×10^5 cms./sec.

$$\text{Hence } W = \frac{(70-10) \times 13.6 \times 981^2}{0.70 \times 1.40 \times 10^5} = 3.28 \times 10^6 \text{ ergs/cm.2/sec.}$$

$$= 0.33 \text{ watts/cm.}^2.$$

Bubbling was obtained, however, as has been calculated above, at a maximum value of power of 0.016 watts/cms.^{2}. Allowing for possible experimental errors and approximations, it can be

concluded that the actual power required was of the order of 0.02 watts per cm.^{2} which is about sixteen times less than the simple theory would anticipate.

The tentative explanation offered that this cavitation effect is a phenomenon produced by the continuous pressure of radiation and not by the alternating or harmonic pressure in the wave, seems to be borne out here.

As proven in the experiments here bubbling occurs under the action of much less energy than simple theory would require for the formation of a bubble directly from the liquid; and if other factors, such as surface tension, etc., be taken into account in the theory, the amount of energy required for cavitation to ensue would be higher rather than lower. It has been pointed out that bubbles collected at the nodal planes of stationary waves, and it was possible to observe bubbles in these planes coalescing and growing larger. It was observed also that bubbles formed more copiously and with less expenditure of energy if the fluid were agitated.

Bubbling has been observed in these experiments in naptha at frequencies as high as 173,000 cycles/sec.; other experimenters have observed bubble formation in water with frequencies as high as 300,000 cycles/sec. It is almost inconceivable that the actual generation of a bubble directly from the liquid state could take place in the time of one alternation at these very high frequencies. (The pressure is a maximum twice during each period.)

Further, as has been mentioned, the time required to

collect a relatively small sample of the gas, although bubbling was copious, was quite long, and the gas when collected was found to consist almost entirely of air.

Taking all of these factors into consideration, it seems reasonable that the type of cavitation which we are dealing with here is not a rupture of the medium due to the harmonic pressure in the wave but an effect due to the existence of small, invisible, bubbles of air or other gas or small bubbles of gas surrounding dust nuclei, already present in the liquid. A gas bubble, having very different elastic properties from the liquid, would act as a good reflector and scatterer of the ultrasonic energy and would be easily dtiven by the pressure of radiation into the nodes. On colliding with one another, or when coming together in the nodal planes, smaller bubbles coalesce to form larger ones. As mentioned before the bubbles were observed to move easily under the radiation pressure. This theory would account for the fact that agitation assisted the bubbling, since agitating the liquid would cause it to take more air in solution or suspension, and would also cause portions of the fluid which had not already given up their small bubbles to come into the energy field.

It is, however, only reasonable to assume that the resultant pressure at a point in the fluid through which a wave passes is reduced for a half period by the wave motion in the way indicated by the simple theory first outlined. This lessening of pressure would have the effect of causing the liquid to evaporate into bubbles already formed, thus causing them to grow in size. When

the bubble passed out of the energy field the naptha gas, except for a small amount due to the vapour pressure, would condense, leaving as a residue only the small air bubble nucleus. This would explain why it required so long to collect an appreciable amount of the gas even though bubbling appeared to be rapid and the bubbles quite large.

Conclusions.

It would seem well to add to the first suggestion of Boyle and Taylor, that the phenomenon is due to a continuous pressure of radiation acting upon small bubbles of gas already in the liquid, by concluding that the phenomenon is due to the combined actions of the continuous pressure of radiation upon small bubbles already present and of the harmonic pressure in the wave acting to cause evaporation of the liquid into bubbles already formed. The time for the action of the harmonic pressure is, of course, extremely short.

The fact that the bubbles did not rise easily from horizontal nodal planes substantiates the conclusions drawn in Addendum III that the pressure of radiation in stationary waves is a harmonic function of the distance from the reflector.

The most important conclusion arising from such experiments is that the formation of bubbles in the path of an ultrasonic beam in a liquid seriously limits the energy transmissible. The production of discontinuities formed by these bubbles, the reflection and scattering of the waves by them, all must act to dissipate some of the energy transmitted; and, as we have seen, at least in volatile liquids, this action occurs at much lower values of emitted energy than simple theory would lead us to anticipate.

Addendum I.

For one of the transmitters a new supply of quartz, in the form of two 10-inch circular mosaics, had been obtained. It was necessary to make a survey of these discs for their piezo-electric quality. Twining of right and left handed quartz crystals often causes a neutralization or a reversal of the sign of the piezo-electric properties.

A "Cremer-Edelmann" string electrometer was set up as is indicated in fig. 9. S is the string, and P_1 and P_2 the poles of the applied electric field. The electromotive force of the battery, A, was 721 volts, and the mid point of the battery was grounded by means of the potentiometer arrangement, B.

Fig. 10 is a sketch giving the dimensions of the test block upon which the quartz was placed. The quartz to be tested was placed at F on the block D, which was insulated from the earth but connected to the string of the electrometer. By means of the lever, G, and weight, H, a pressure was applied to the quartz, which produced a charge on each surface. The key K_2, fig. 9, was closed, thus grounding the testblock and letting the charge run to earth. The key K_2 was then opened, the lever lifted and a charge appeared on the quartz due to this release of pressure. In this way the test block and string of the electrometer became charged. The electric field between the poles caused the string to deflect by an amount, at the middle of the string, proportional to the potential. The deflection was read on a scale in the eyepiece of a microscope attached to the

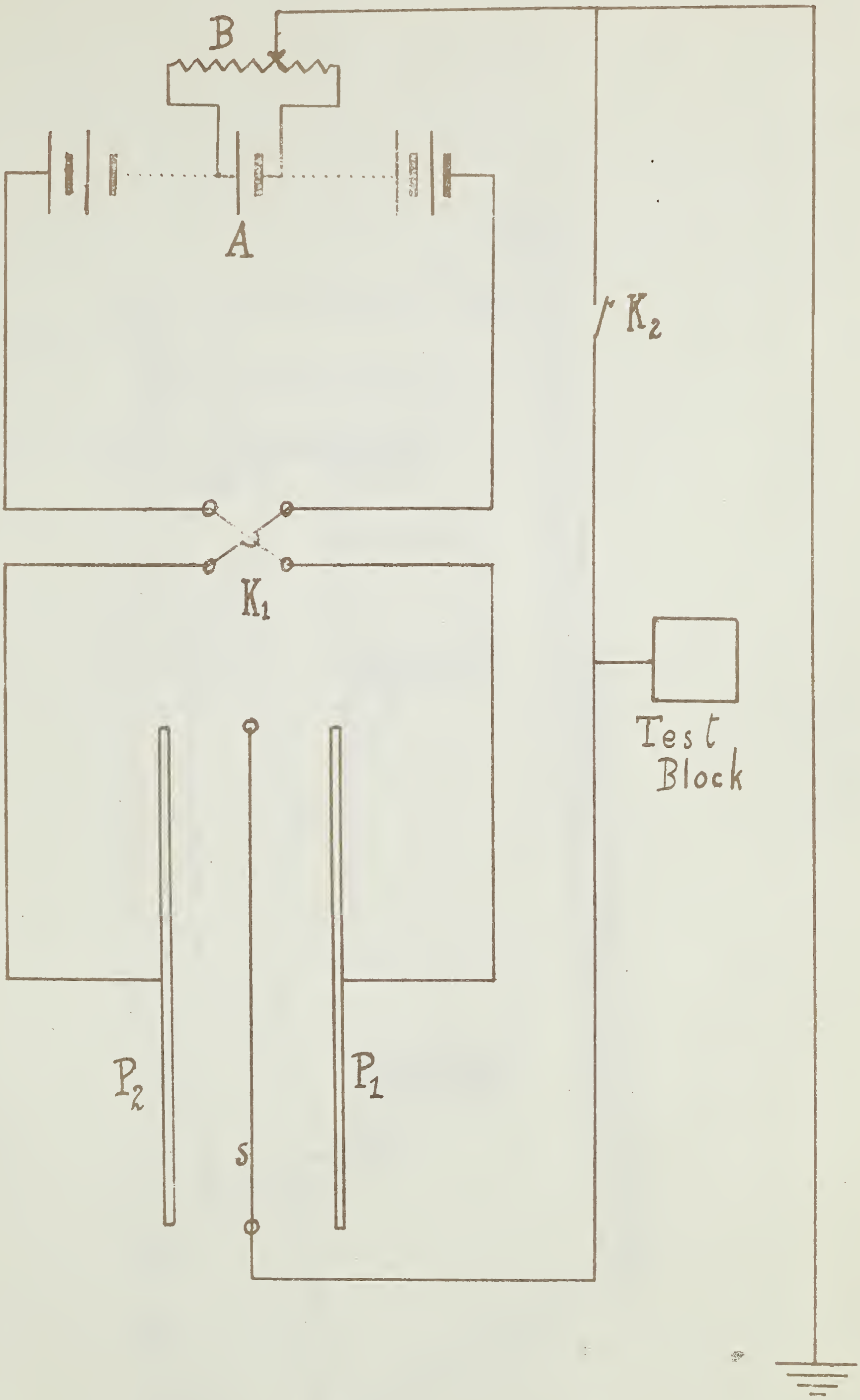

Fig 9.

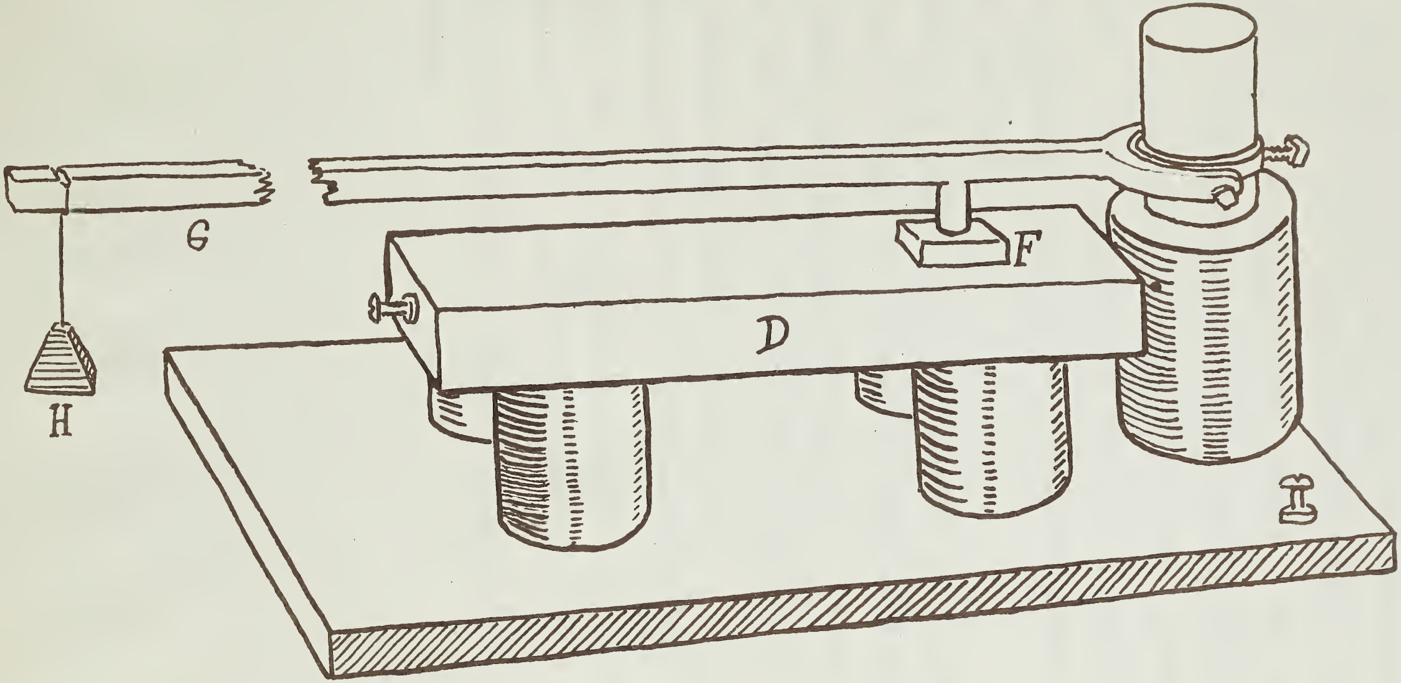

Fig. 10.

electrometer.

The piece of quartz under test would be placed on the block at F, fig. 10, and a reading of the string deflection taken. The field between P_1 and P_2 would then be reversed by the key K_1, fig. 9, and another reading taken. The average of these deflections was taken as the result. For comparison purposes, a standard piece of quartz whose quality was known was used, this standard having a piezo-electric quality of 25 arbitrary units. This standard was used as the test piece frequently during measurements in order to be sure that the apparatus gave consistent results.

These tests might have been carried out by grounding the block when the weight was lifted and measuring the deflection of the string upon imposing the pressure. It was found, however, that more definiteness and hence more accuracy of reading resulted if the readings were taken upon release of pressure.

Each square inch of two quartz mosaics, 0.50 cms. in thickness and 25 cms. in diameter, was tested, and figs. 11 and 12 show diagrams of the mosaics with the quality of the quartz indicated in the squares in the arbitrary units adopted for the standard.

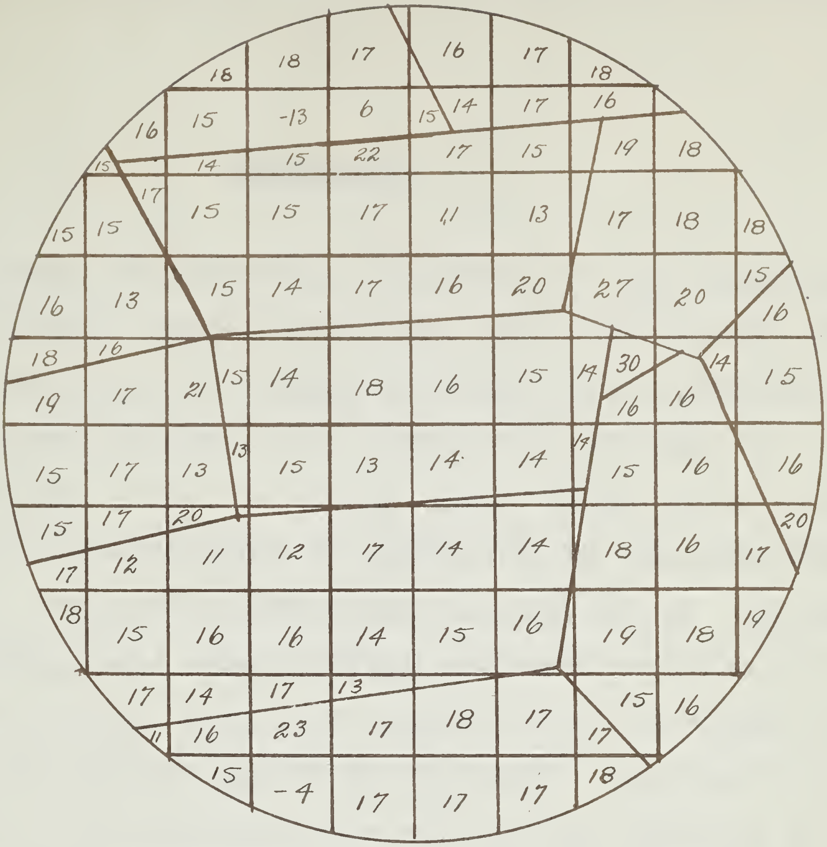
18 18 17 16 17 18
16 15 -13 6 15 14 17 16
15 14 15 22 17 15 19 18
15 15 17 15 15 17 11 13 17 18 18
16 13 15 14 17 16 20 27 20 15 16
18 16 19 17 21 15 14 18 16 15 14 30 16 16 14 15
15 17 13 13 15 13 14 14 14 15 16 16
15 17 20 17 12 11 12 17 14 14 18 16 20 17
18 15 16 16 14 15 16 19 18 19
17 14 17 13 15 16 11 16 23 17 18 17 17
15 -4 17 17 17 18

Fig. 11

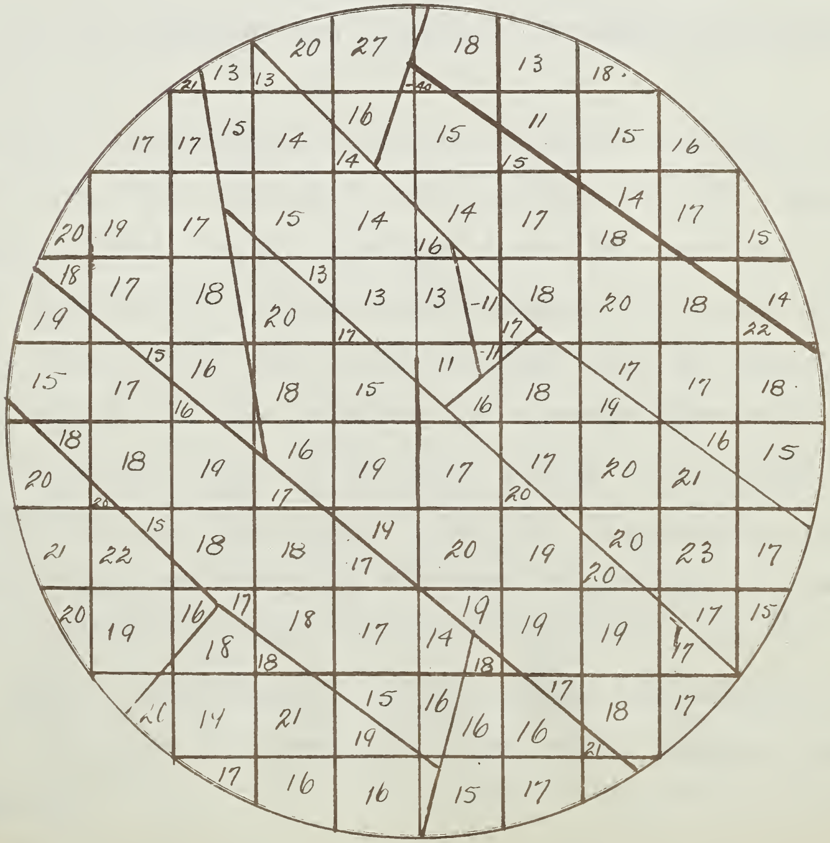
20 27 18 13 18
21 13 13 -40
17 17 15 14 16 15 11 15 16 14 15
20 19 17 15 14 14 17 14 17 18 15 16
18 17 18 13 20 13 13 -11 18 20 18 14 22 19 17
15 16 15 17 16 18 15 11 -11 16 18 17 17 18 19
18 20 18 19 16 19 17 17 20 16 15 21 17 20
20 21 22 15 18 18 14 17 20 19 20 23 17 20
20 19 16 17 18 18 17 19 14 19 19 17 15 17
18 18 20 14 21 15 19 16 16 16 17 18 17 21
17 16 16 15 17

Fig. 12

Addendum II.

Incidental to the work on Cavitation, for the purpose of finding the most suitable transmitter with which to work, and its best frequencies for energy emission, a short investigation of the resonant frequencies of different transmitters had to be carried out. This was done by taking "energy characteristic" curves of the transmitters, or portions of the characteristic curves, at frequencies around their resonant points. The pendulum readings were taken for these various frequencies, the transmitter voltage being kept constant, and the readings so obtained were plotted against the frequency. When the transmitter came into mechanical resonance with the frequency of the electrical oscillations imposed upon it, the energy emission was very greatly increased and the pendulum reading increased accordingly.

Consider a double-steel-plate transmitter. Let the thickness of the quartz be negligible in comparison with the thickness of the steel plates. In the simplest theory the two steel plates act together in a manner very analogous to an open organ pipe. The steel plates will be in resonance with the vibrations of the quartz, if the time required for the vibration to travel from the outer face of one of the steel plates to the outer face of the other and back again is one half the period, or any odd number of half periods; i.e. the steel plates are in resonance when their combined thickness is an odd multiple of quarter wavelengths. Results previously obtained by Boyle and Lehmann+ have shown this to be the case except when the quartz becomes appre-

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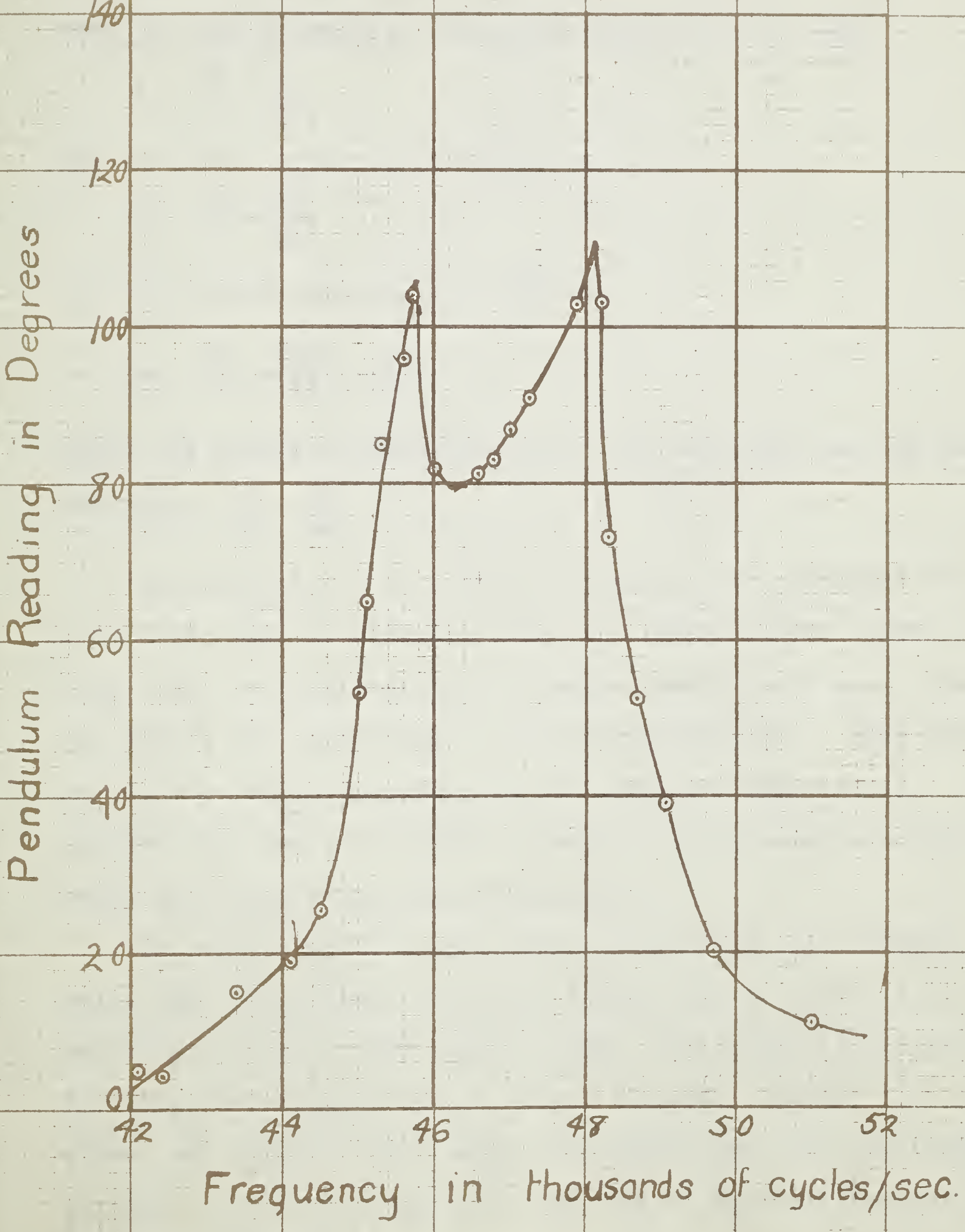
+ Loc. Cit. 1.

CROSS SECTION 10 X 10

Fig 6



ciably thick in comparison with a wave length.

Let V = the velocity of propagation of the wave in the steel, λ = the wave length and n the frequency. Let d = the thickness of the steel plate.

Then for the fundamental resonant point, $d = \frac{\lambda}{4}$, and

$$n_o = \frac{V}{\lambda} = \frac{V}{4d}.$$

For the first harmonic, $d = \frac{3\lambda}{4}$, and

$$n_1 = \frac{V}{\lambda} = \frac{3V}{4d},$$

For the second harmonic, $d = \frac{5\lambda}{4}$, and

$$n_2 = \frac{V}{\lambda} = \frac{5V}{4d}\ ;\ \text{etc.}$$

Hence the resonant frequencies, n_o, n_1, n_2, etc. are in the ratio $\frac{V}{2d} : \frac{3V}{2d} : \frac{5V}{2d} \ldots\ldots\ = 1 : 3 : 5 \ldots\ldots$

Transmitter No. 5, a double-steel-plate instrument having a 10 inch diameter circular face, two steel plates 2.125 and 2.127 cms. in thickness, and a quartz mosaic 0.50 cms. thick, was set up and oscillated at various frequencies. Characteristic curves were taken to determine its resonant frequencies. Fig. (6) plotted from Table III shows a section of the characteristic curve about the fundamental frequency.

It will be noticed that there were really two resonant peaks near each other, each peak being due, no doubt, to the resonance of one of the steel plates. The peaks do not coincide for various reasons. One plate was slightly thicker than the other; one radiated into water, the other into the insulating mixture. The effect may have been due to the oscillations in the electrical circuit.

The resonant frequencies obtained experimentally from this transmitter were:

n_o = 48000 cycles per second, approximately.

n_1 = 164700 cycles per second.

n_2 = 291800 " " "

n_3 = 424500 " " "

n_4 = 560000 " " "

Table III.

Frequency in cycles per second	Pendulum Reading in degrees	Frequency in cycles per second	Pendulum Reading in degrees.
47900	103.0	46600	81.6
46800	83.1	45000	53.6
47250	91.0	45200	74.5
45300	85.0	48250	103.0
46000	82.0	49100	39.2
44100	19.2	44500	25.8
43400	15.3	42100	5.0
42450	4.2	45100	65.0
47000	86.7	45600	96.0
48700	52.4	45700	104.0
48300	73.0	51000	11.0
49700	20.0		

Assuming that the resonant frequencies are in the ratio 1, 3, 5, 7 etc., but that there is an end correction due to the presence of the quartz (whose thickness in comparison with a wave length is not negligible), the fundamental frequency would be given by any of the following relations: $\frac{n_1 - n_o}{2}$, $\frac{n_2 - n_1}{2}$,

$\frac{n_3 - n_2}{2}$, $\frac{n_4 - n_3}{2}$, $\frac{n_2 - n_o}{4}$, $\frac{n_3 - n_1}{4}$, $\frac{n_4 - n_2}{4}$, $\frac{n_3 - n_o}{6}$ and $\frac{n_4 - n_1}{6}$, $\frac{n_4 - n_o}{8}$.

An average of the above values was taken and was found to be 64490 cycles/sec., making an end correction of approximately 64490 - 48000 = 16490 cycles/sec. As the frequency increases, however, the thickness of the quartz becomes greater in comparison with a wave length and the end correction should increase. Using the above end correction throughout, the true frequencies for resonance in cycles/sec. become:

n_o = 64490

n_1 = 181190

n_2 = 308290

n_3 = 440990

n_4 = 576490,

and their ratios are 1.03 : 2.90 : 4.92 : 7.05 : 9.20. These ratios are based on the average of n , $n_1/3$, $n_2/5$, $n_3/7$ and $n_4/9$. This is in fair agreement with the 1, 3, 5, 7, 9, etc., sequence. It will be noticed that the ratios become greater than the 1, 3, 5, 7, series as the frequency increases, which shows, as pointed out above, that the end correction increases with the frequency.

Addendum III.

It was desired to study the pressure of radiation in stationary waves produced by reflection. A low resonant frequency is necessary in order that the wave lengths should be large enough to be examined conveniently.

Transmitter No. 6, a pile-of-plates instrument having three steel plates and two quartz discs, was set up in the large tank, fig. 2, and operated at a frequency, n, of 30350 cycles per second. This type of instrument was used since it gave a lower fundamental resonant frequency than the double-steel-plate type for the same thickness of plates. The plates used in the transmitter were the thickest which could be obtained.

The velocity of sound in water, V, is 1.48×10^5 cms. per second. Hence the wave length, λ, $= \frac{V}{n} = \frac{1.48 \times 10^5}{30350} = 4.88$ cms.

Stationary waves were produced by reflecting the incident energy from a steel plate, about 20 inches square and 7/8 inch thick, placed parallel to the face of the transmitter.

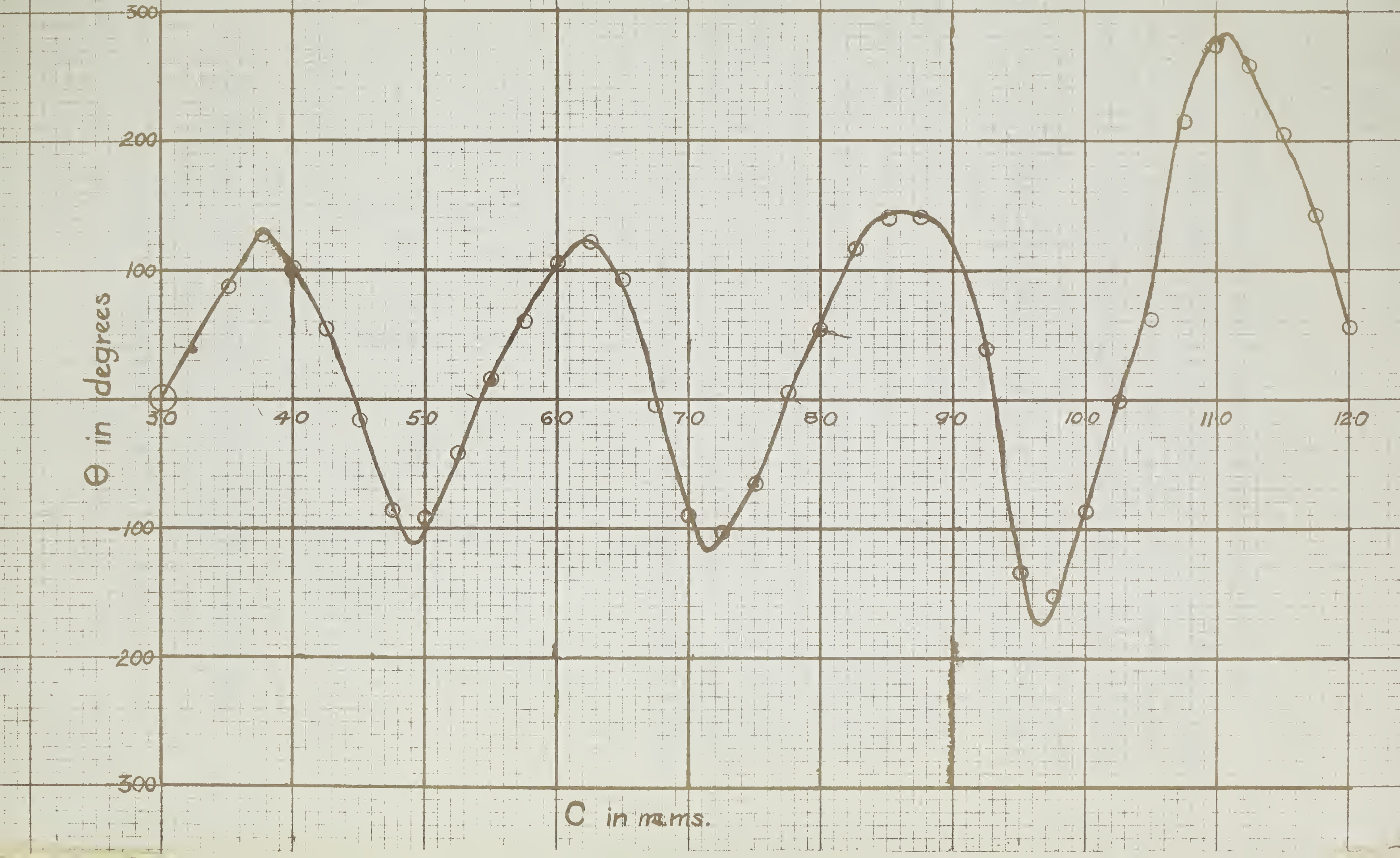
A single vane air pendulum of diameter 1.0 cm., suspended by 0.0015 cm. phosphor bronze strip, was placed in the field. Pendulum readings were taken at various distances from the transmitter. Table IV gives the pendulum readings with the corresponding relative distances from the transmitter, and the readings are plotted against the distance in fig. 7. Positive readings are those when the pendulum was deflected away from the transmitter and negative when the deflection was toward the transmitter.

It will be observed from the curve that the maximum positive readings occur one-half wave length apart as do also the maximum negative readings, and that the pressure of radiation is zero at positions one quarter wave length apart.

TABLE IV.

Relative distance from transmitter, d, in cms.	Pendulum reading, θ, in degrees.	Relative distance from transmitter, d in cms.	Pendulum reading, θ, in degrees.
3.50	87.0	8.50	739.0
3.75	128.0	8.75	141.0
4.00	100.0	9.00	169.0
4.25	54.0	9.25	38.0
4.50	-17.0	9.50	-135.0
4.75	-87.0	9.75	-153.0
5.00	-91.0	10.00	-88.0
5.25	-43.0	10.25	-2.0
5.50	12.9	10.50	63.0
5.75	59.5	10.75	215.0
6.00	105.5	11.00	274.0
6.25	121.2	11.25	258.0
6.50	92.7	11.50	204.0
6.75	-4.9	11.75	140.4
7.00	-88.8	12.00	58.0
7.25	-102.5	12.25	-46.8
7.50	-65.0	12.50	-39.0
7.75	4.0	12.75	-1.0
8.00	52.0	13.00	-20.0
8.25	117.0	12.60	-21.0

Fig. 7


300
200
100
-100
-200
-300
Θ in degrees
3.0
4.0
5.0
6.0
7.0
8.0
9.0
10.0
11.0
12.0
C in m.ms.

Several similar sets of readings were taken each set yielding about the same results as the typical case cited here.

In order to locate the positions of zero pressure in the stationary wave field with respect to the nodes and antinodes, a tray was placed just in front of the reflector and the cinder dust method, refered to before, was used to obtain a picture of the stationary waves. The cinders sank in curtains at the nodes. A double-vane lead pendulum was suspended just above the tray and moved over the field to determine the positions of zero readings and positive and negative readings. It was found that the readings were zero at both the nodes and the antinodes and were positive on the sides of the nodes nearer the transmitter and negative on the other sides of the nodes. Below, in fig. 8 is shown the sign of the readings in the field. The heavy lines are the nodes.

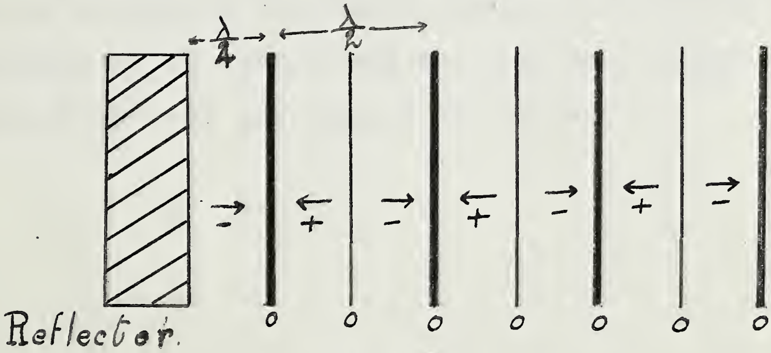


Fig. 8.

It was noticed that a small amount of the cinder dust collected in lines at the antinodes. Whether this was due to the presence of the first harmonic, or to the fact that the pressure of radiation at the antinodes was zero, it was not possible to determine.

There may be a doubt as to these results since the sound waves were not quite plane and since the disturbance of the field caused by the pendulum might have been enough to affect the measurements. The diameter of the pendulum was, however, only about one fifth of a wave length. Further work will be necessary with a still longer wave length.

On the whole it can be concluded that the pressure of radiation in the stationary waves produced under the present conditions is an alternating function of the distance along the direction of propagation of the incident waves, and that it is zero at the nodes and antinodes in the stationary-wave field.

It will be noticed that this conclusion is supported by the fact, mentioned in Part I, that bubbles, produced by cavitation, in the stationary-wave field produced by reflection at the surface of the liquid, tended to remain in layers parallel to the surface and one half wave length apart. The reasons for this are given fully in Part I.

Part II.

Ultrasound readily lends itself to an experimental study of the relations existing between the amount of sound energy reflected by a reflector and the thickness of the reflecting body; whereas sound of ordinary pitch would present many and almost insuperable difficulties in such a problem.

The method followed in this investigation and described in detail below was originated by Boyle and Lehmann[18]. They, however, investigated the relation only over a range of one half wave length in the reflecting material, and used only one material. In the present experiments the study has been advanced over a somewhat wider range both in thicknesses and in materials, and the relations were checked at different frequencies.

The problem is to determine the ratio of reflected to incident sound energy of a continuous train of plane waves of sound incident normally upon a partition. The theoretical relations given here are taken from a paper by Boyle and Rawlinson[19], giving the full details of a mathematical solution suggested in the theory of the phenomenon by Lord Rayleigh[20]. The mathematical problem there treated may be enunciated as follows:

A train of plane waves of sound travelling in an infinite, homogeneous medium is incident normally upon a parallel-faced partition of homogeneous material placed in the medium. The

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[18] Paper in press.

[19] A paper not yet published.

[20] "Theory of Sound", Lord Rayleigh, Vol. II, p. 86.

partition is infinitely extended but of finite thickness. What proportion of the incident energy is reflected?

The result obtained is expressed in the following relation:

$$R = \frac{\left(\frac{V\rho}{V_1\rho_1} - \frac{V_1\rho_1}{V\rho}\right)^2}{4\cot^2\frac{2\pi L}{\lambda_1} + \left(\frac{V\rho}{V_1\rho_1} + \frac{V_1\rho_1}{V\rho}\right)^2} \quad \ldots\ldots\ldots\ldots\ldots \quad (4)$$

where R is the ratio of incident to reflected energy density,

V = the velocity of sound in the incident medium,

V_1 = " " " " " " partition,

ρ = the density of the incident medium,

ρ_1 = " " " " Partition,

L = the thickness of the partition, and

λ_1 = the wave length in the partition.

By the principle of the conservation of energy the ratio of the energy transmitted to the incident energy must be 1 - R.

Consider equation (4).

If $L = \frac{n\lambda_1}{2}$ where n is an integer, then $\frac{2\pi L}{\lambda_1} = n\pi$ and $\cot\frac{2\pi L}{\lambda_1} = \pm\infty$. Hence $R = 0$.

If $L = (2n + 1)\frac{\lambda_1}{4}$ where n is an integer, $\frac{2\pi L}{\lambda_1} = (2n + 1)\frac{\pi}{2}$ and $\cot\frac{2\pi L}{\lambda_1} = 0$. Hence R is a maximum.

The solution indicates, therefore, that if the partition be of thickness zero or an integral number of half wave lengths the reflection will be a minimum and the transmission a maximum; and if the thickness be an odd number of quarter wave lengths the reflection will be a maximum and the transmission a minimum.

The relation also shows that, other things being equal, reflection will be greatest when $\frac{V\rho}{V_1\rho_1} - \frac{V_1\rho_1}{V\rho}$ is greatest, i.e. when $V\rho$ differs greatly from $V_1\rho_1$. The reflection will be zero when $V\rho = V_1\rho_1$, a fact already made use of in previous papers on the reflection of ultrasonics[21]. This relation applies to all thicknesses of partition, and it should hold for all frequencies.

It might be noticed that the reflection is least and transmission greatest when the thickness of the partition is correct for the partition to vibrate longitudinally in resonance with the incident wave train, i.e. when its thickness is an integral number of half wave lengths.

The above is an outline of the mathematical theory to test which this research was carried out.

When the propogating medium is a liquid or a solid perfect reflection should almost invariably occur from a partition consisting of a layer of gas. Let us take the case of a water medium and a partition of air, such as may be formed by enclosing a layer of air between two thin sheets of mica, and consider the amount of reflection from the partition as reflector. If the mica is very thin in comparison with a quarter wave length its presence may be neglected, and even if it did reflect to an appreciable amount it would only increase the efficiency of the reflector.

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[21] Boyle and Taylor - Trans.Roy.Soc.Can., XIX, III, pp.197-203

The velocity of sound in air, V_1, = 0.344×10^5 cms./sec.at 20°C.

The density of air, ρ_1, = 0.00114 grams/c.c. at 20°C.

The velocity of sound in water, V, = 1.48×10^5 cms./sec.[22]

The density of water, ρ, = 1 gram/c.c.

Hence for perpendicular incidence,

$$R = \frac{\left(\dfrac{1.48 \times 10^5 \times 1}{0.344 \times 10^5 \times 0.00114} - \dfrac{0.344 \times 10^5 \times 0.00114}{1.48 \times 10^5 \times 1}\right)^2}{4 \cot^2 \dfrac{2\pi L}{\lambda_1} + \left(\dfrac{1.48 \times 10^5 \times 1}{0.344 \times 10^5 \times 0.00114} + \dfrac{0.344 \times 10^5 \times 0.00114}{1.48 \times 10^5 \times 1}\right)^2}$$

$$= \frac{1.04 \times 10^7}{4 \cot^2 \dfrac{2\pi L}{\lambda_1} + 1.04 \times 10^7} \quad \text{very approximately.}$$

From this it follows that 90% of the incident energy is reflected when R = 0.9, i.e. when

$$4 \cot^2 \frac{2\pi L}{\lambda_1} = \frac{1.04 \times 10^7}{0.9} - 1.04 \times 10^7 = 0.12 \times 10^7.$$

This occurs when $L = 0.0008 \dfrac{\lambda_1}{2}$.

Therefore all values of thickness which differ from an exact half wave length of the sound in air by more than four-ten thousandths of a wave length would reflect more than ninety per cent of the incident energy. The wave length, in air, of the ultrasonic waves employed in the following experiments was of the order of 0.1 cms. Hence the range in which less than 90% would be reflected is less than 8×10^{-5} cms. at each half wave length. That a partition could be made with a thichness which lies within this range for more than a very limited area

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22. Boyle and Taylor, Trans.Roy.Soc.Can. XIX, III, pp. 197-203.

of the surface is exceedingly improbable. Hence in the following work where it was required to measure energies by a standard torsion pendulum, from the vane of which the reflection could be considered perfect, an air vane pendulum was used. Such pendulums are made by sealing hermetically very thin sheets of mica over a light circular aluminum ring, thus enclosing a thin layer of air.

Although the vane of a torsion pendulum is restricted in area, and cannot be regarded as an indefinitely extended partition, nevertheless it can be made to yield information on the theory outlined above, and to a great extent put the salient results of the theory to experimental test.

A test in the relation of equation 4 is possible by taking readings of pendulums of various thicknesses when they are subjected to the radiation of an ultrasonic beam under exactly similar conditions. The pendulum reading is approximately proportional to 2 RE (see section on torsion pendulums, P. 7), and thus, as long as the energy emission of the transmitter remains constant, the pendulum reading will be proportional to R.

The heavier a pendulum, the greater its moment of inertia and its period, and hence the longer is the time for it to come to rest after having been deflected. Hence a list of possible convenient materials for experiment was made and the product of the density and velocity of sound computed for each. This product is proportional to the product of the density and

Table V.

Substance	Modulus of Elasticity or Young's Modulus in dynes/cms.^{2}	Density in grams/c.c.	Velocity of Sound $V = \sqrt{E/\rho}$ cms/sec.	Relative mass per wave-length ρV ($\propto \rho\lambda$).
Paraffin	0.115 x 10^{11}	0.909	1.304 x 10^5	1.18 x 10^5
Water	0.220 "	1.00	1.43 "	1.48 "
Gypsum	3.53 "	2.32	1.23 "	2.86 "
Sulphur	0.913 "	1.9	2.19 "	4.17 "
Magnesium	4.32 "	1.74	4.98 "	8.67 "
Antimony	1.67 "	6.6	1.59 "	10.5 "
Tin	5.29 "	2.5	4.60 "	11.5 "
Glass	6 "	2.8 (approx.)	4.6 "	13 "
Marble	6.75 "	2.5-2.8	6.5 "	13.4 "
Lead	1.70 "	11.3	1.23 "	13.8 "
Duralumin	7.4 "	2.79	5.16 "	14.4 "
Bismuth	3.14 "	9.8	1.79 "	17.5 "
Cadmium	4.12 "	8.6	2.19 "	18.8 "
Brass	10.7 "	8.44	3.50 "	29.5 "
Nickel	17.6 "	8.7	4.50 "	39.1 "
Steel	14.9 "	7.83	4.99 "	39.1 "

wave length, which product determines the mass of the pendulum. The results of this survey are given in Table V.

A glance at Table V will show the materials most suitable for use. Any material with product $V\rho$, greater than lead would be inconveniently heavy in these experiments. Metallic magnesium would have been suitable but it was impossible to obtain it in time. After considering the availability and ease of working of the different materials, lead, duralumin and paraffin were selected. The lead and duralumin were well separated in the metals and had high reflecting power, and paraffin was included as being a substance possessing such elastic qualities that its reflecting power must be very low.

A number of pendulums of various thicknesses were made from each of the materials named. It was, of course, necessary to work under steady conditions of emission and it was desirable to have a field as nearly uniform as could be obtained. It was also desirable to work at a fairly high frequency in order that the pendulums would not become too heavy before the desired number of wave-lengths thickness was obtained.

Transmitter No. 2, a double-steel-plate instrument, was examined to determine how well it adapted itself to these requirements. This transmitter had the smallest radiating face of any of the instruments at hand, and consequently [23] the beam was of greater width, at any given frequency, than any of the other transmitters would have given.

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[23] Ibid. p. 5

By taking a "characteristic" emission curve of the transmitter two very suitable frequencies of resonance were discovered at 531,200 and 307,200 cycles per sec. It was necessary to work at a resonant frequency in order to obtain sufficient intensity. At these frequencies the characteristic curve was not sharp, but nicely rounded, so that the emission would change only very slightly for small changes in frequency. Moreover, at these frequencies, the wave meter showed that any electrical harmonics present were not of appreciable magnitude.

Although the ratio of reflected to incident energy density does not depend upon the way the intensity of the incident beam varies over the surface of the reflector, it was deemed advisable to have a field as nearly uniform as possible. Consequently both vertical and horizontal cross sections of the beam were taken and the pendulum placed in the centre of the beam. The area of the pendulum was such that the intensity, over its surface was nearly constant. Fig. 13 shows the cross section of the field at 531,200 cycles/sec. with the pendulum vane fitted into it. This curve was plotted from pendulum readings given in Table VI.

Lead.

The transmitter was set to oscillate at a frequency of 531,200 cycles per sec. A series of lead pendulums, the counter balance type[24], of different thicknesses were suspended by the same suspension, 0.003 Phosphor Bronze wire, in~~to~~ the same position in the beam, and the pendulum readings taken. After

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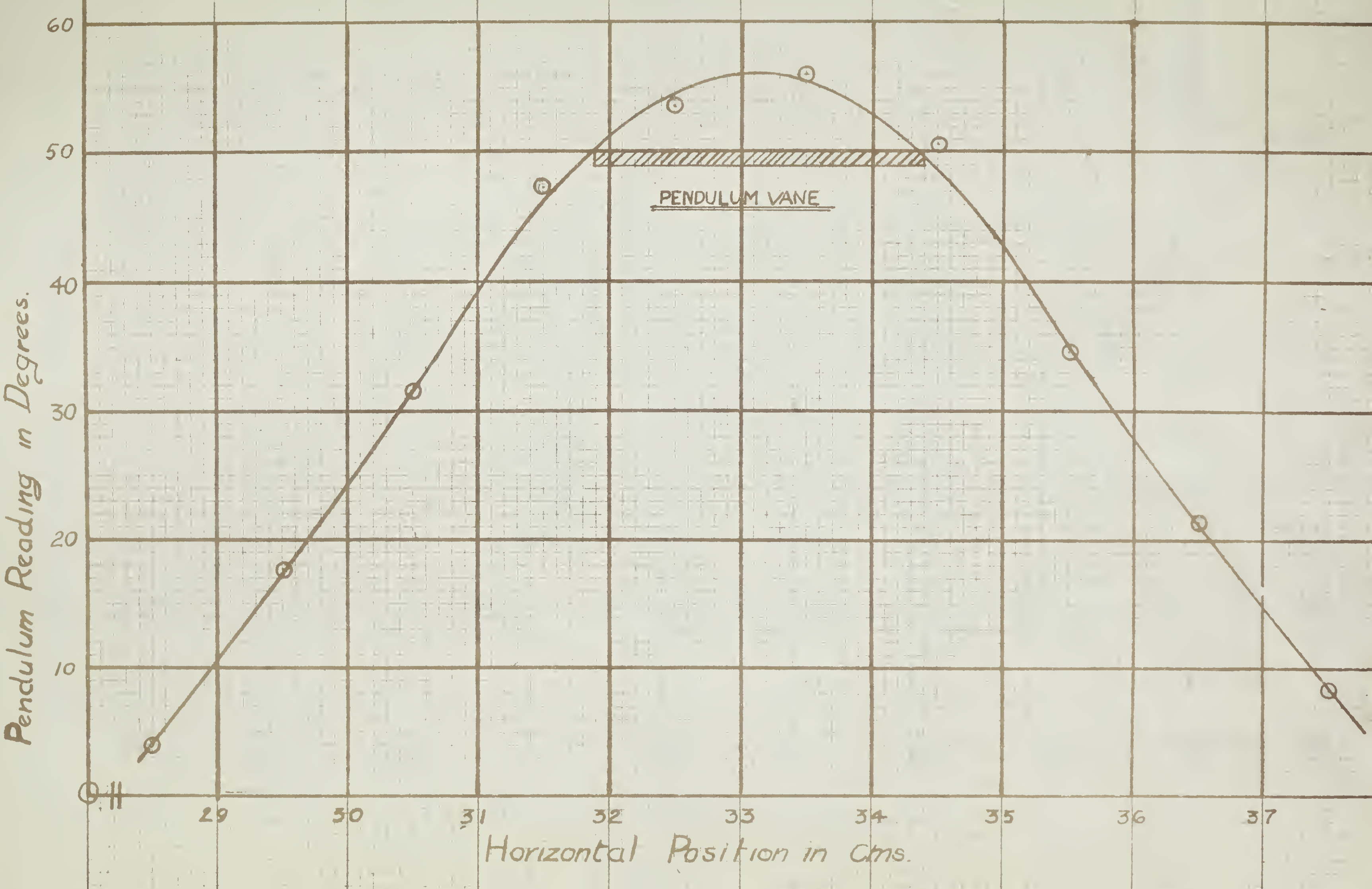
[24] Ibid. p. 5

Fig no. 13.


PENDULUM VANE
Pendulum Reading in Degrees.
60
50
40
30
20
10
0
Horizontal Position in Cms.
29
30
31
32
33
34
35
36
37

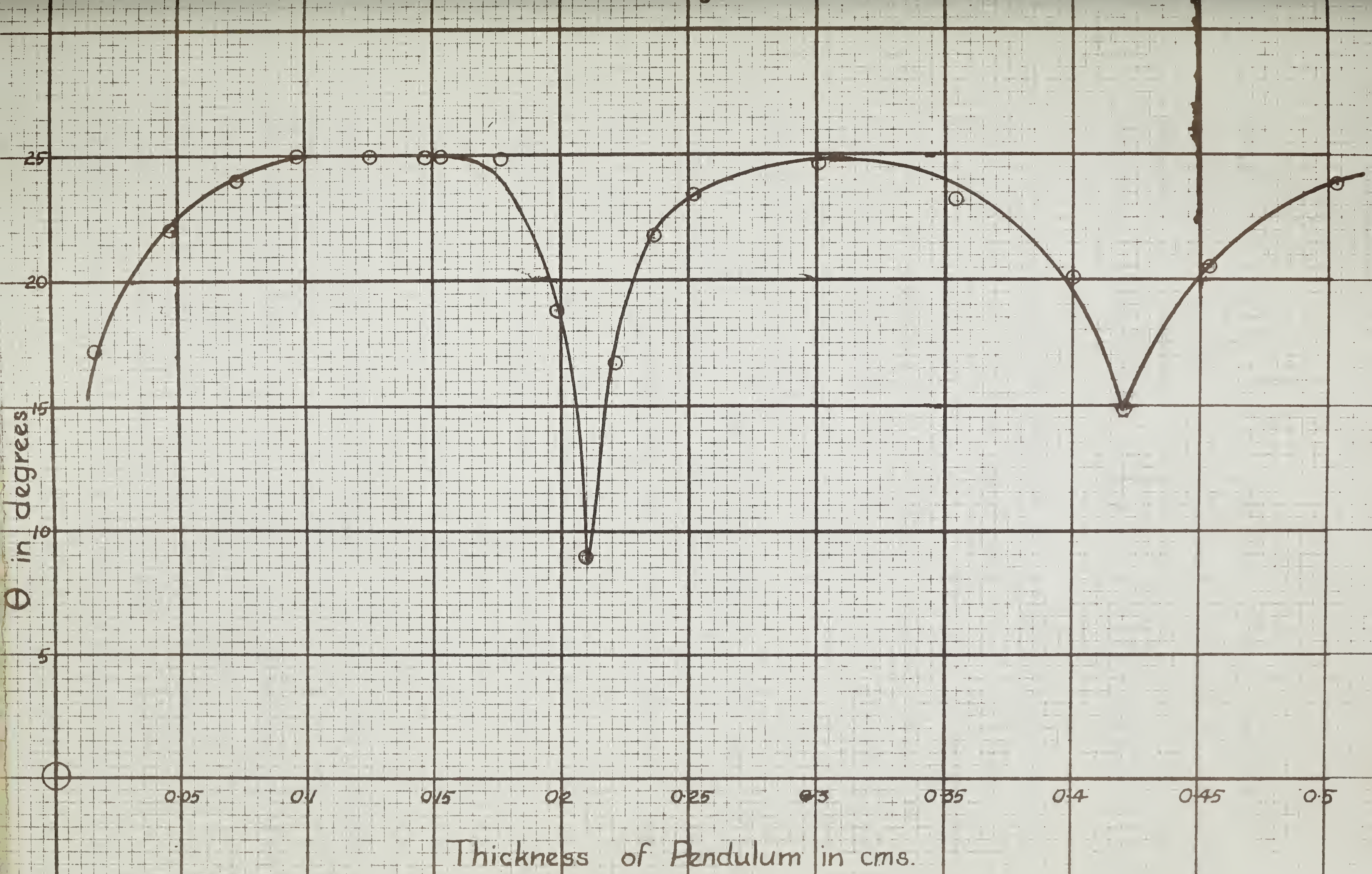

Θ in degrees
25
20
15
10
5
0·05
0·1
0·15
0·2
0·25
0·3
0·35
0·4
0·45
0·5
Thickness of Pendulum in cms.

Fig. 14.

every two or three readings some one pendulum, used as a standard, was replaced in the beam to determine if the energy emission of the transmitter were constant. If it were not a correction could be made in the ratio of the readings of the standard pendulum.

The pendulum readings with the corresponding thickness of pendulum vane are given in Table VII and the readings plotted against the thickness in fig. 14. There are two well marked dips in the curve where the reflection ratio is very low. By considering the theory outlined above it is seen that these dips occur at exact half wave lengths of the ultra sound in the material of the pendulum.

It should be noticed that the effect of the area of the counter weight exposed to the radiation, was only a few percent, i.e. within the experimental error, of the effect of the area of the vane.

Another method of experiment to determine the half wavelength and the velocity was carried out as follows:- A pendulum of the same material giving the maximum value of deflection on the axis of the beam was used to take a "characteristic" curve of the transmitter over a range of frequency near the operating frequency of 531,200 cycles per second. That is, the pendulum reading was taken for various frequencies near 531,200 cycles/sec. at the same transmitter voltage as in the previous experiment, and the deflection plotted against the frequency. The readings are given in Table VIII and plotted against frequency in fig. 15 (curve 1). Then the pendulum which gave the point nearest the bottom of the first dip in the deflection-against-thickness

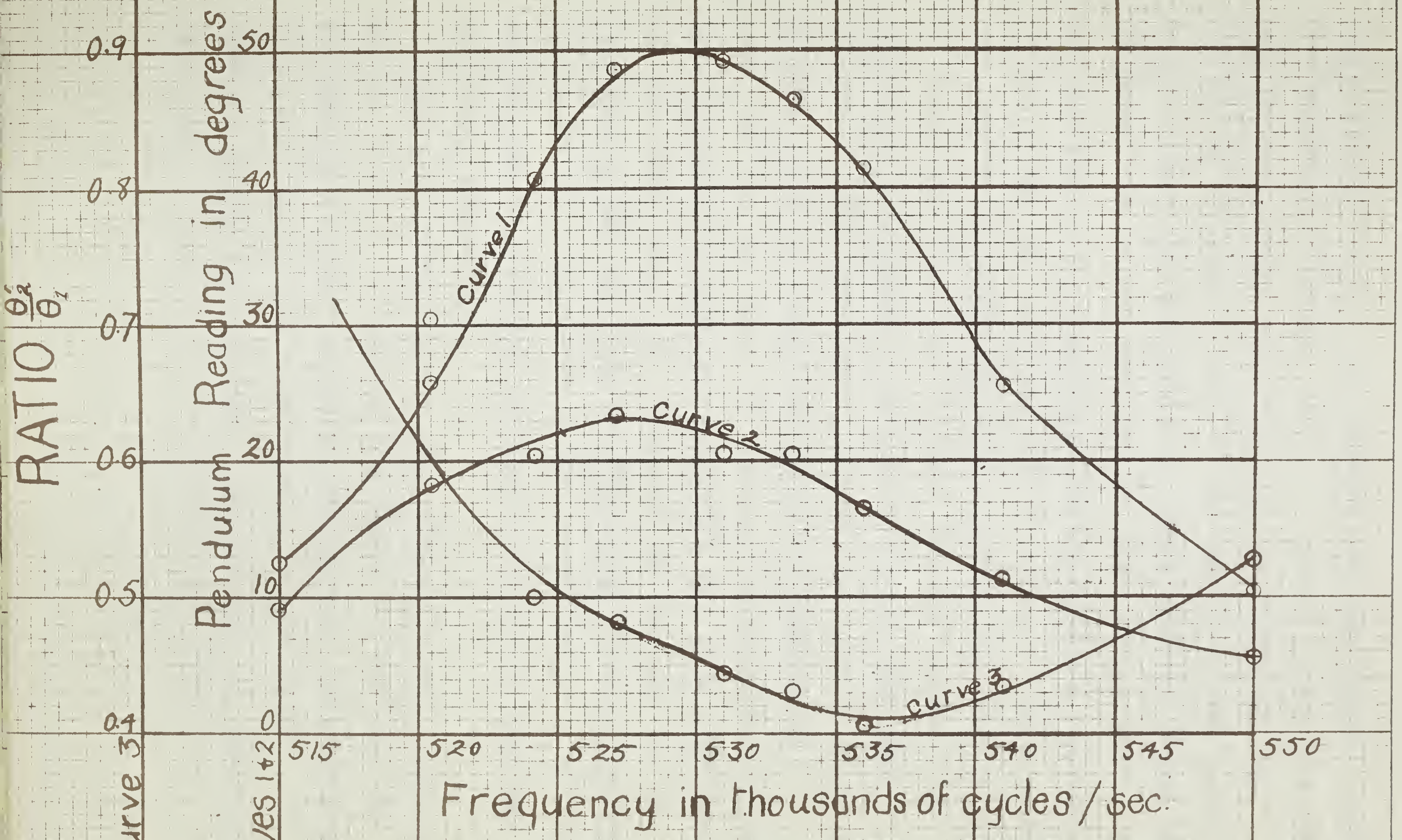

RATIO $\frac{\theta_2'}{\theta_1'}$
0.9
0.8
0.7
0.6
0.5
0.4
Curve 3
Pendulum Reading in degrees
50
40
30
20
10
0
Curves 1+2
515
520
525
530
535
540
545
550
Frequency in thousands of cycles / sec.
Curve 1
Curve 2
Curve 3

Fig. 15

curve, fig. 12, that is the pendulum which was nearest a half wave length in thickness, was used to obtain a characteristic over the same range, Table VIII, fig. 15, (curve 2). By varying the frequency a value of the exact frequency could be found at which this pendulum is an exact half-wave length in thickness, for at such frequency the ratio of the reading of this pendulum to the readings of the one giving the maximum reading at the same frequency will be smallest. These ratios are given in Table VIII and plotted against frequency in fig. 15 (curve 3). From this it appears that the minimum ratio occurred at a thickness in lead of 0.210 cm., from which it follows that the wave-length was 0.420 cms. at a frequency of 536,000 cycles per second; and hence the velocity was 2.23×10^5 cms. per sec. This value of the velocity is near that determined by Boyle and Lehmann[25], viz. 2.06×10^5 cms./sec., and much higher than value quoted for the velocity for low frequency sound, (viz. 1.23×10^5 cms./sec.) as determined by static methods. The result of this experiment is an effective confirmation of the result obtained by the use of pendulums of various thicknesses.

The theoretical curve of the reflection ratio R against fig. 16, curve 1, was plotted from Table IX. The value for the velocity of sound in lead as determined by the experiments just described was used in calculating the values of R. It might be pointed out here that, theoretically, and neglecting absorption, the pressure on the face of the pendulum when it

Fig:

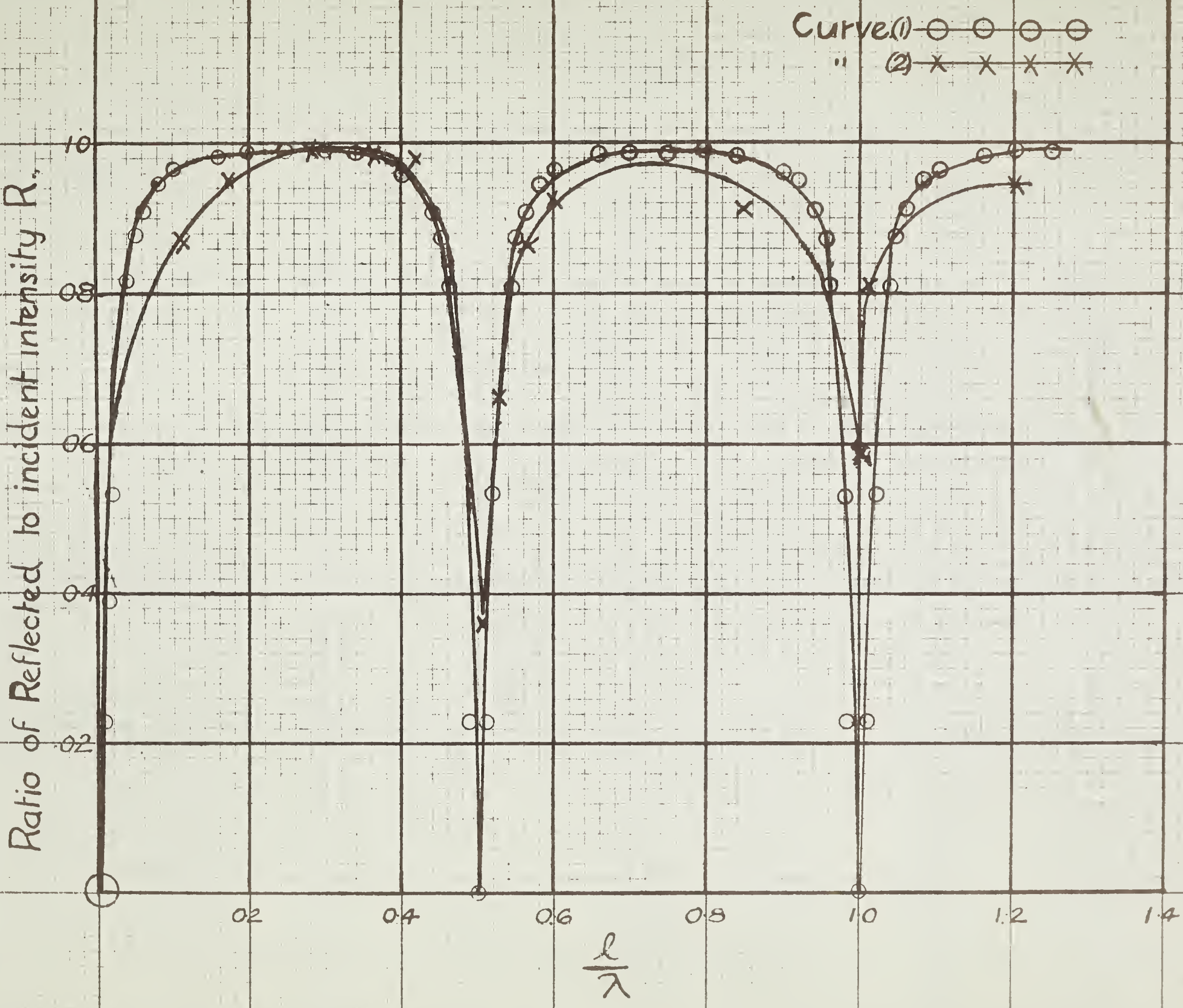

Curve (1) ○ ○ ○ ○
" (2) × × × ×
Ratio of Reflected to incident intensity R.,
10
08
06
04
02
02
04
06
08
10
12
14
$\frac{l}{\lambda}$

Fig. 16

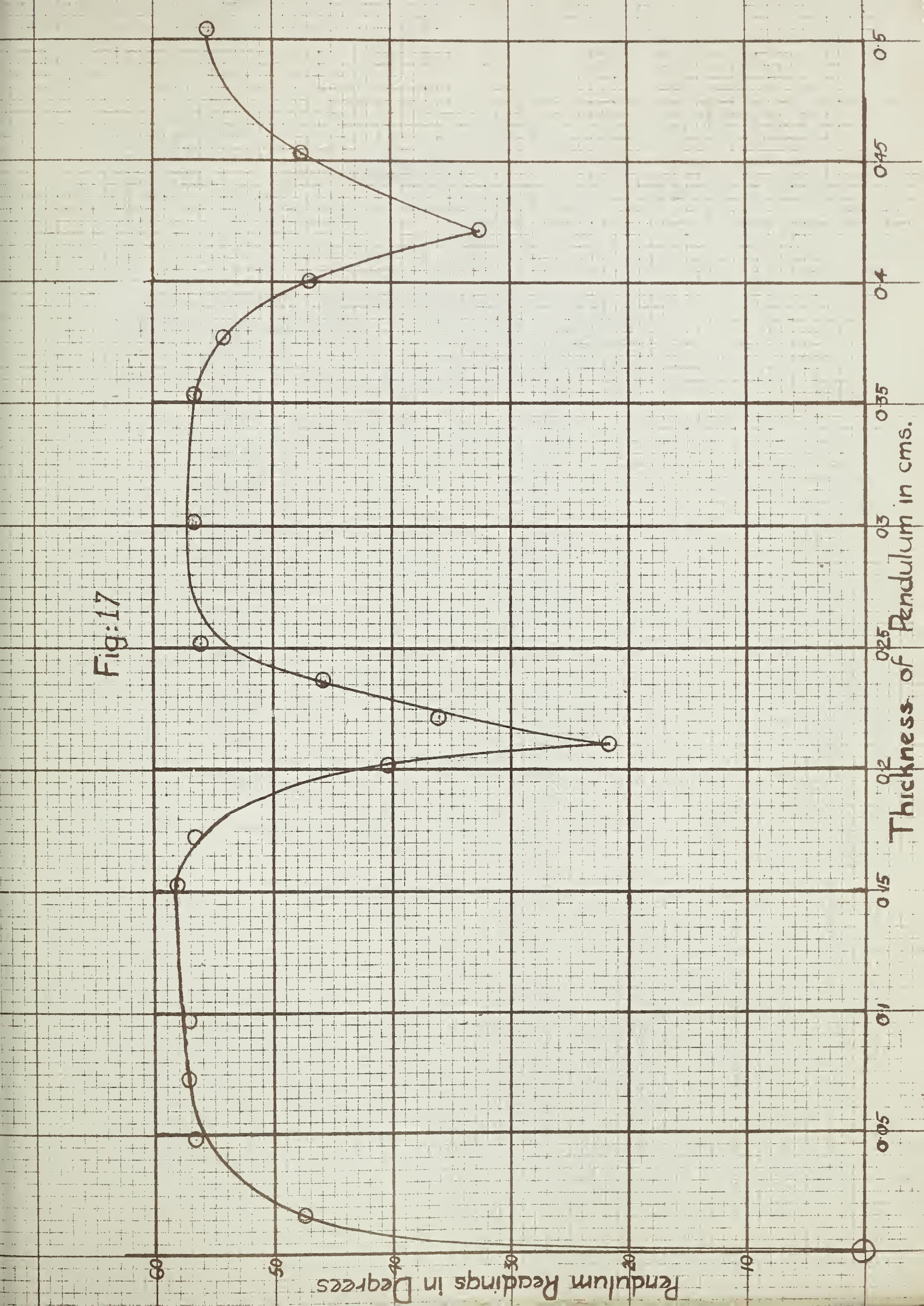


Fig: 17

Pendulum Readings in Degrees
60
50
40
30
20
10
Thickness of Pendulum in cms.
0·05
0·1
0·15
0·2
0·25
0·3
0·35
0·4
0·45
0·5

Table VI.

Horizontal Position in cms.	Pendulum Reading in degrees.
28.5	4.0
29.5	17.7
30.5	31.5
31.5	47.3
32.5	53.6
33.5	56.0
34.5	50.6
35.5	34.6
36.5	21.3
37.5	8.3

is an integral number of half wave lengths in thickness is exactly equal to and balanced by the back pressure on the back of the pendulum due to transmitted energy. But if there is any absorption, and there is certain to be some, the transmitted energy will not be as great as the incident, and therefore the pendulum reading will not be zero. Hence, in any curve obtained by this method, zero reading at the integral numbers of half wave lengths cannot be expected.

Using the highest reading obtained to correspond to the maximum value of R, the values of R, corresponding to various values of the pendulum readings ~~which had been observed~~, were computed, Table X, and plotted in fig. 16, curve 2, in order to compare the results of the experiment with the theory, curve 1.

As a check another series of lead pendulums was taken at the same frequency with a 0.0025 cm. phosphor bronze wire suspension. The pendulum readings were plotted against the thickness in fig. 17 ~~from the experimental values~~, Table XI. It is seen that this curve is very nearly a repetition of the one taken with the 0.0030 cm. phosphor bronze wire and that it gives the same value for the wave length.

Another series of experiments with lead vanes was carried out at a different frequency, viz. 307,200 cycles per second. The suspension used was 0.0025 cm. phosphor bronze wire. The thicknesses of the vanes with the corresponding readings are given in Table XII. As before, characteristics were taken with two vanes, one giving the maximum, the other the minimum reading in the series, and the ratio of the readings at various

frequencies plotted against the frequency to obtain the exact half wave length and the velocity. The results are given in Table XIII. These curves are not shown since they are similar to those at 531,200 cycles per second and have no intrinsic interest. As before the pendulum readings were reduced to reflection ratios and plotted against $\frac{1}{\lambda_1}$ in fig. 16, curve 3, from Table XIV. The velocity determined by these experiments was 2.22 x 10^5 cms. per sec.

Table VII.

Thickness of Pendulum in cms.	Pendulum Reading, θ, in degrees	Thickness of Pendulum in cms.	Pendulum Reading, θ, in degrees
0.016	17.2	0.152	25.0
0.097	25.0	0.210	8.9
0.048	22.0	0.420	14.8
0.074	24.0	0.236	21.8
0.125	24.9	0.152	25.0
0.148	24.9	0.400	20.1
0.176	24.8	0.453	20.5
0.198	18.8	0.503	23.8
0.221	16.7	0.355	23.1
0.252	23.4	0.301	24.6

Table VIII.

Frequency in cycles/sec.	Reading of vane giving maximum reading, θ_1 in degrees	Reading of vane near $\frac{\lambda_1}{2}$ in thickness, θ_2 in degrees	Ratio of readings, $\frac{\theta_2}{\theta_1}$
515000	12.8	9.0	0.704
520500	26.0	18.3	0.705
524300	40.8	20.4	0.500
527200	48.8	23.5	0.482
531000	49.5	20.8	0.444
533500	46.5	20.5	0.430
536000	41.2	16.6	0.403
541000	25.6	11.1	0.435
550000	10.5	5.5	0.524

Table IX.

Velocity of Sound in lead, V_1, = 2.33×10^5 cms./sec.

" " " " water, V, = 1.48×10^5 cms./sec.

Density of lead, ρ_1, = 11.3 grams/cm.^{3}

" " water, ρ, = 1 gram/cm.^{3}

$$R = \frac{\left(\frac{V\rho}{V_1\rho_1} - \frac{V_1\rho_1}{V\rho}\right)^2}{4\cot^2\frac{2\pi L}{\lambda_1} + \left(\frac{V\rho}{V_1\rho_1} + \frac{V_1\rho_1}{V\rho}\right)^2}.$$

L/λ_1	$4\cot^2\frac{2\pi L}{\lambda_1}$	R	L/λ_1	$4\cot^2\frac{2\pi L}{\lambda_1}$	R
0.00	∞	0.000	0.25	0	0.987
0.01	1011	0.222	0.30	0.422	0.986
0.015	450	0.389	0.34	1.61	0.980
0.02	251	0.532	0.40	7.62	0.961
0.025	159	0.639			
0.03	110	0.717	0.45	37.9	0.874
0.04	60.7	0.817			
0.05	37.9	0.874	0.47	110	0.717
0.06	25.6	0.906	0.475	159	0.639
0.08	13.2	0.944	0.48	251	0.532
0.10	7.62	0.961	0.485	450	0.389
0.16	1.61	0.980	0.49	1011	0.222
0.20	0.422	0.986	0.50	∞	0.000

Table X.

Maximum pendulum reading, 25^0, corresponds to maximum value of R (0.987) in the theoretical curve, λ_1 as computed = 0.420 cms.

Thickness of Pendulum, L, in cms.	L/λ_1	Pendulum in degrees, θ^0.	R ($= \theta \times \frac{0.987}{25}$)
0.016	0.0381	17.2	0.679
0.097	0.231	25.0	0.987
0.048	0.114	22.0	0.869
0.074	0.176	24.0	0.948
0.125	0.298	24.9	0.984
0.148	0.352	24.9	0.984
0.176	0.419	24.8	0.980
0.198	0.472	18.8	0.742
0.211	0.527	16.7	0.660
0.252	0.600	23.4	0.925
0.152	0.362	25.0	0.987
0.210	0.500	8.9	0.352
0.420	1.000	14.8	0.584
0.236	0.562	21.8	0.861
0.152	0.362	25.0	0.987
0.400	0.954	20.1	0.795
0.453	1.080	20.5	0.810
0.503	1.200	23.8	0.940
0.355	0.845	23.1	0.912
0.310	0.717	24.6	0.972

Fig: 16, Curve 3

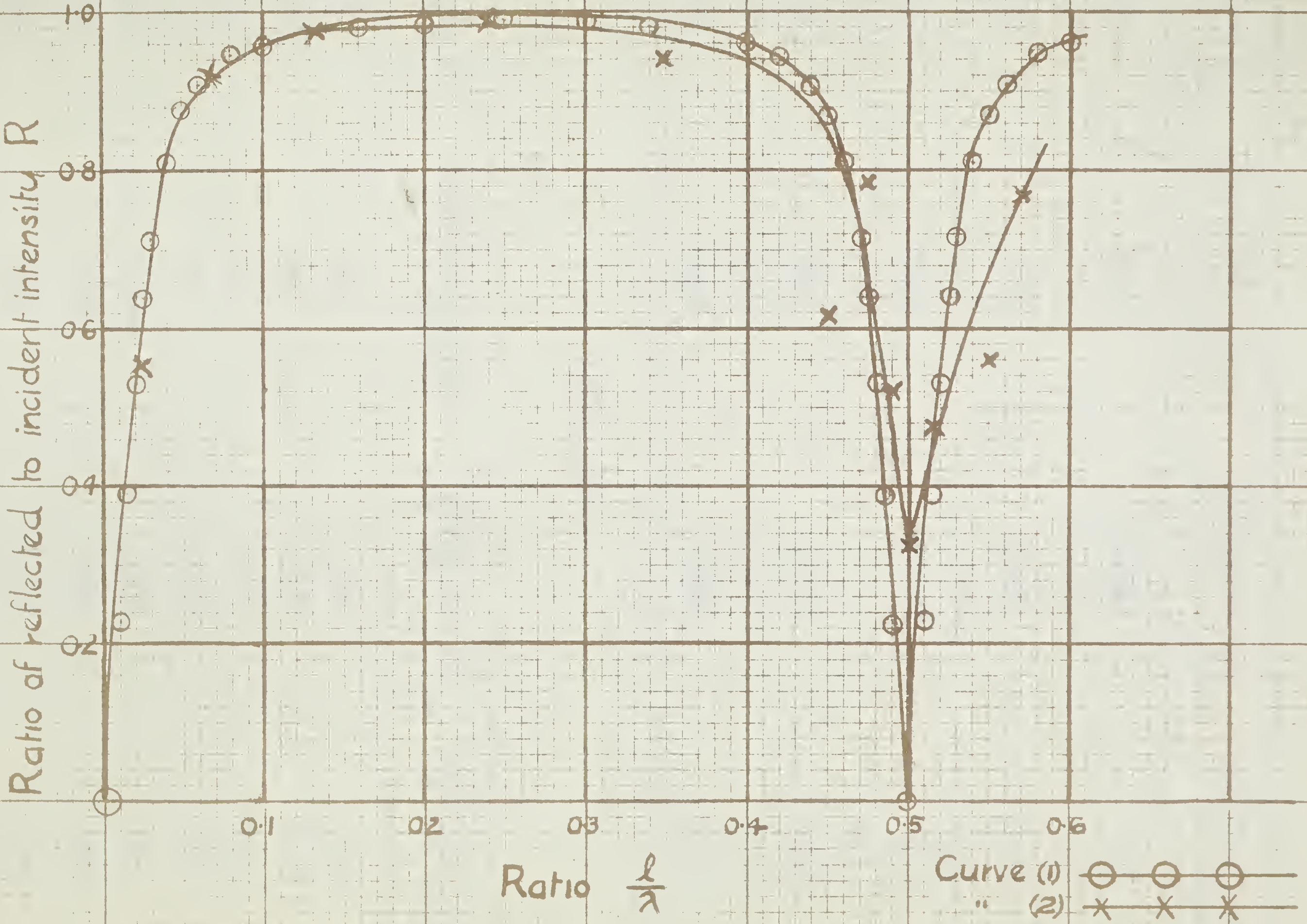


Table XI.

Thickness of Pendulum in cms.	Pendulum reading, θ, in degrees	Thickness of Pendulum in cms.	Pendulum reading θ, in degrees
0.152	58.0	0.354	56.5
0.210	21.8	0.175	56.7
0.236	45.6	0.097	57.0
0.202	40.2	0.074	57.1
0.422	32.6	0.048	56.8
0.453	47.5	0.016	47.5
0.503	55.1	0.252	56.0
0.400	46.9	0.301	56.4
0.222	36.0	0.378	53.8

Table XII.

Thickness of Pendulums in cms.	Pendulum reading, θ, in degrees	Thickness of Pendulums in cms.	Pendulum reading, θ, in degrees.
0.176	47.7	0.016	26.5
0.355	25.1	0.047	44.2
0.378	22.9	0.097	47.7
0.301	37.9	0.332	29.8
0.400	29.6	0.367	15.7
0.252	40.5	0.252	45.3
0.502	40.8	0.419	37.5

Table XIII.

Frequency in cycles/sec.	Reading of Vane giving maximum reading, θ, in degrees.	Reading of vane near $\lambda/2$ in thickness, θ'	Ratio of readings, θ'/θ
306500	54.0	14.3	0.265
308200	54.5	14.1	0.261
303800	38.0	9.2	0.242
301900	25.7	6.1	0.230
300500	17.8	4.04	0.224
300000	15.3	4.04	0.262

Table XIV.

Maximum pendulum reading, 47.7°, corresponds to maximum value of R (0.987) in the theoretical curve. λ_1 as computed = 0.734 cms.

Thickness of pendulum, L, in cms.	L/λ_1	Pendulum reading, θ, in degrees.	$R\ (= \theta \times \frac{0.987}{47.7})$
0.176	0.240	47.7	0.987
0.355	0.484	25.1	0.520
0.378	0.516	22.9	0.475
0.301	0.476	37.9	0.785
0.400	0.546	26.9	0.557
0.016	0.0218	26.5	0.549
0.047	0.0641	44.2	0.917
0.097	0.132	47.7	0.987
0.332	0.453	29.8	0.617
0.367	0.501	15.7	0.325
0.252	0.344	45.3	0.940
0.419	0.572	37.5	0.777

Duralumin.

The entire process was repeated using another material, viz. duralumin, instead of lead. Below in Table XV the different sets of readings which were taken, and the ratios which were computed from them, are named with the numbers of the tables which follow, in which are recorded the data obtained, and the numbers of the corresponding figures and curves. Only those curves of intrinsic

interest are shown, the others are similar to the corresponding curves in the lead series already shown and have been used for computation only.

Table XV.

Readings taken or Ratios computed	Table	Fig.	Curve
Series taken at 531200 cycles/sec. Suspension 0.0030 cm. P.B. Wire	XVI		
Characteristic about 531200 cycles/sec. with vane giving maximum reading. (Same curve as for lead used)	XVII		
Characteristic about 531200 cycles/sec. with vane giving lowest reading in first dip.	XVII		
Ratio of readings of second to readings of first characteristic	XVII		
The theoretical curve, R against L/λ_1, using the value of the velocity determined from the ratios of Table XIV	XVIII	18	1
The readings at 531200 cycles/sec. reduced to ratios and plotted against L/λ_1	XIX	18	2
Check series at 531200 cycles/sec. Suspension 0.0025 cm. P.B. Wire	XX	19	
Series taken at 307200 cycles/sec. Suspension 0.0025 cm. P.B. Wire	XXI		
Characteristic about 307200 cycles/sec. with vane giving maximum reading. (Same curve as for lead used)	XXII		
Same characteristic with pendulum giving lowest reading in the first dip	XXII		
Ratio of readings of second to first characteristics	XXII		
The deflections at 307200 cycles/sec. reduced to ratios and plotted against L/λ_1	XXIII	18	3

Table XVI.

Thickness of Pendulum in cms.	Pendulum reading in degrees	Thickness of Pendulum in cms.	Pendulum reading in degrees
0.405	26.0	0.896	24.9
1.302	19.7		
1.252	11.6	0.802	24.3
1.202	18.5	1.002	24.0
0.496	24.6	1.499	25.7
0.549	18.2	0.052	18.1
0.598	16.7	0.090	23.6
0.652	19.2	0.203	27.0
0.699	20.1	0.300	26.9
		1.145	23.2

Table XVII.

Frequency in cycles/sec.	Reading of Vane giving maximum reading in degrees	Reading of vane near in thickness	Ratio of readings
551000	9.7	7.8	0.790
545200	17.4	8.0	0.460
540000	27.5	12.2	0.444
536500	39.0	19.5	0.500
533000	47.5	25.3	0.533
531600	49.0	25.5	0.521
529000	49.8	31.4	0.631
526200	46.5	29.6	0.637
522000	31.3	23.0	0.725
516800	15.5	13.8	0.891

Table XVIII.

The velocity of sound in duralumin, V_1, = 6.48 x 10^5 cms./sec.

" " " " " water, V, = 1.48 x 10^5 cms./sec.

The density of duralumin, determined experimentally,

$$\rho_1 = 2.81 \text{ grams./cm.}^3.$$

" " " water, ρ = 1.0 gram/cm.^{3}.

$$R = \frac{\left(\frac{V\rho}{V_1\rho_1} - \frac{V_1\rho_1}{V\rho}\right)^2}{4\cot^2\frac{2\pi L}{\lambda_1} + \left(\frac{V\rho}{V_1\rho_1} + \frac{V_1\rho_1}{V\rho}\right)^2}$$

L/λ_1	$4\cot^2\frac{2\pi L}{\lambda_1}$	R	L/λ_1	$4\cot^2\frac{2\pi L}{\lambda_1}$	R
0.00	∞	0.000	0.20	0.422	0.974
0.01	1011	0.128	0.25	0.000	0.977
0.015	450	0.248	0.30	0.422	0.974
0.02	251	0.370	0.34	1.61	0.968
0.025	159	0.479	0.40	7.62	0.931
0.03	110	0.569	0.45	37.9	0.784
0.04	60.7	0.700	0.47	110	0.569
0.05	37.9	0.784	0.475	159	0.479
0.06	25.6	0.838	0.48	251	0.370
0.08	13.2	0.900	0.485	450	0.248
0.10	7.62	0.931	0.49	1011	0.128
0.16	1.61	0.968	0.50	∞	0.000

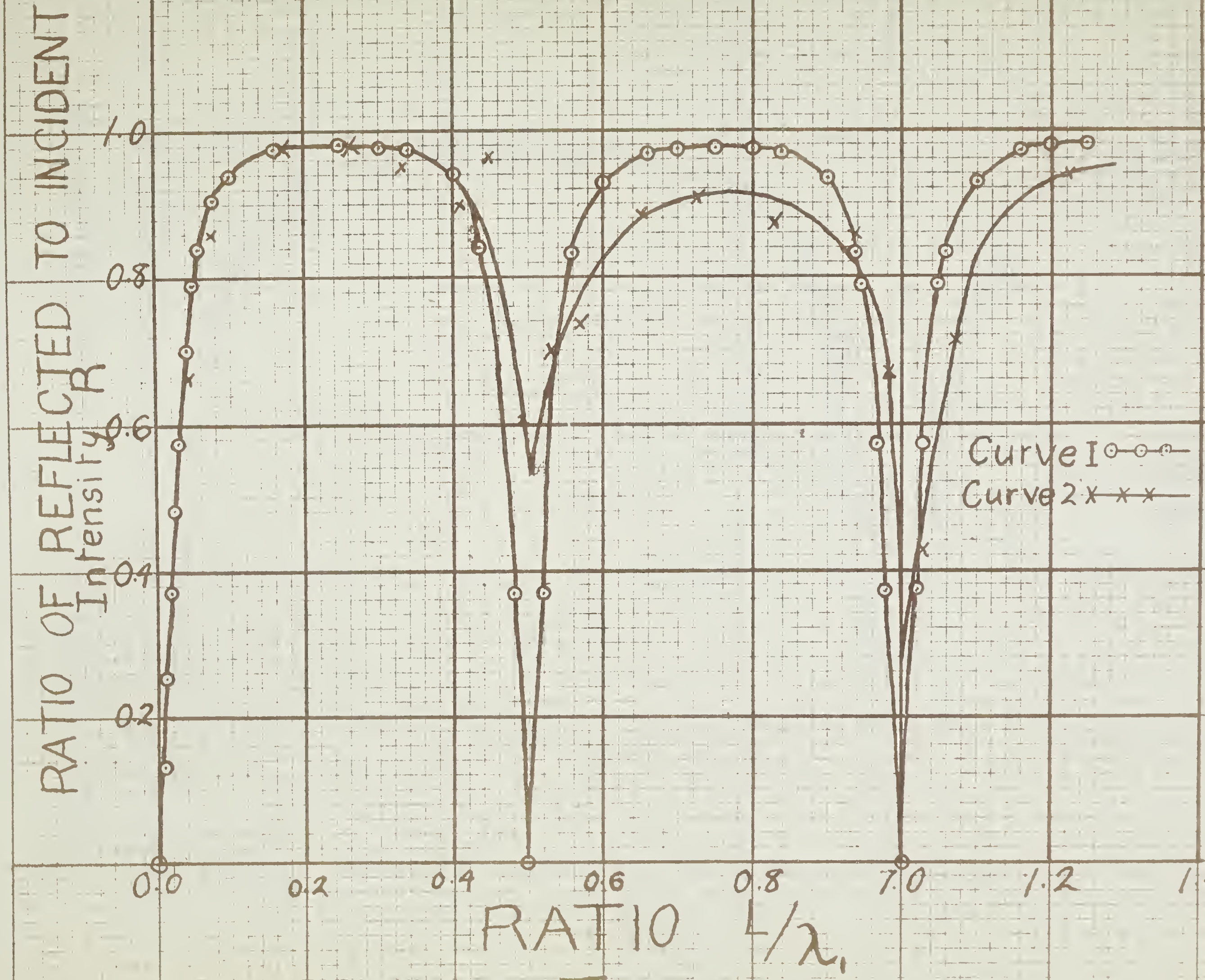


Fig. 18. Curves 1 and 2

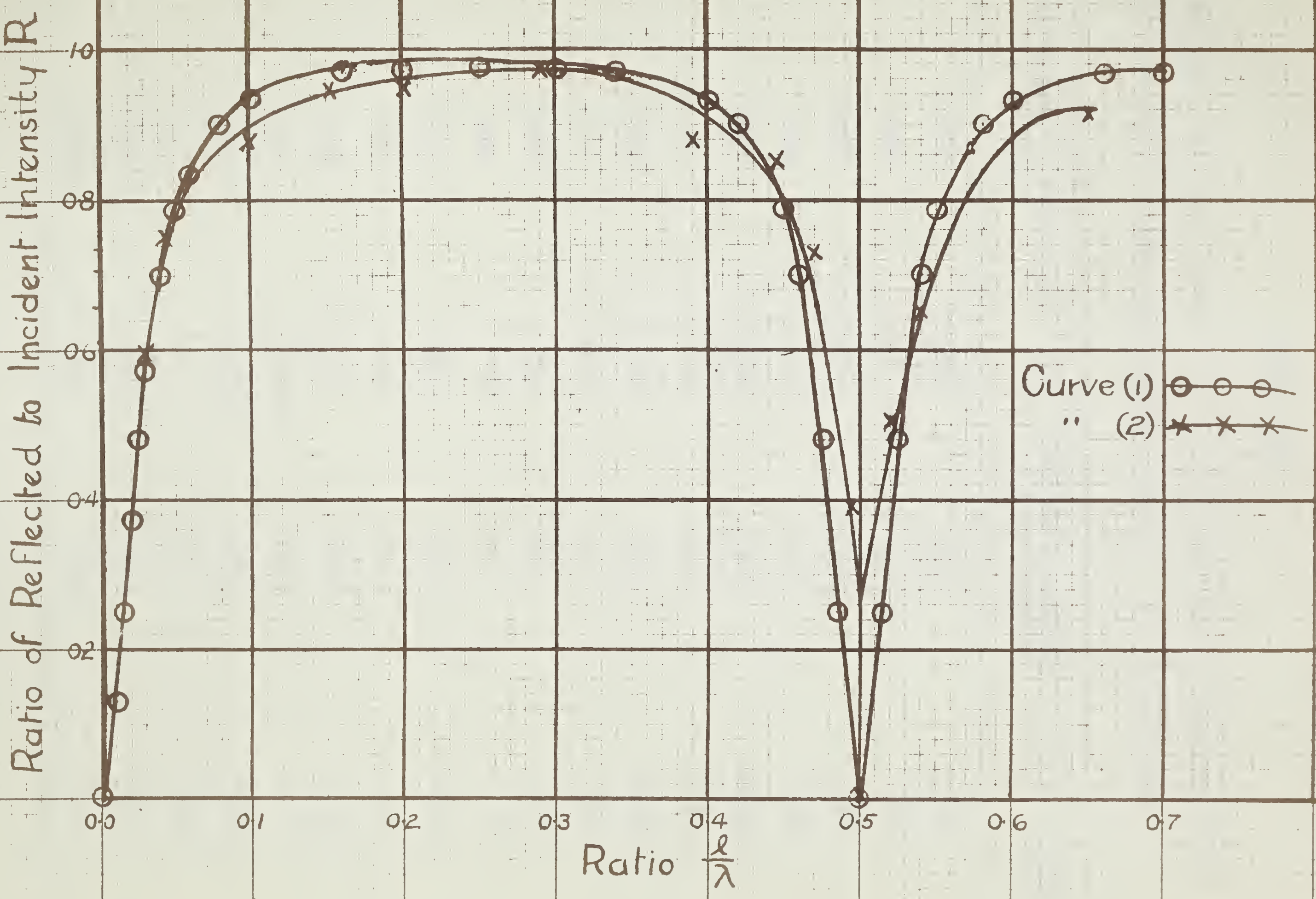


Fig: 18, Curves 1 and 3.

CROSS SECTION 10 X 10

Table XIX.

Maximum pendulum reading, 26.9^0 corresponds to maximum value of R, 0.977, in the theoretical curve. λ_1 = 1.22 cms.

Thickness of Pendulum, L, in cms.	L/λ_1	Pendulum reading θ, in degrees	R $\left(= \theta = \frac{0.977}{26.9}\right)$
0.405	0.332	26.0	0.945
1.302	1.07	19.7	0.716
1.252	1.03	11.6	0.421
1.202	0.987	18.5	0.672
0.496	0.406	24.6	0.894
0.549	0.450	18.2	0.961
0.598	0.490	16.7	0.606
0.652	0.535	19.2	0.698
0.699	0.574	20.1	0.730
0.896	0.735	24.9	0.905
0.802	0.657	24.3	0.884
1.002	0.836	24.0	0.872
1.499	1.23	25.7	0.934
0.052	0.0426	18.1	0.657
0.090	0.0738	23.6	0.858
0.203	0.166	27.0	0.977
0.300	0.246	26.9	0.977
1.145	0.939	23.2	0.843

Table XX.

Thickness of Pendulum in cms.	Pendulum reading, θ, in degrees	Thickness of Pendulum in cms.	Pendulum reading, θ, in degrees.
1.302	43.5	0.547	34.3
0.299	55.3	0.650	37.2
1.252	25.2	0.495	50.4
1.202	48.4	0.698	47.0
0.597	26.5	0.052	39.3
1.500	52.1	0.090	46.9
1.400	51.1	1.144	49.5
1.002	54.7	0.403	53.0
0.802	54.7	0.202	53.7

Table XXI.

Thickness of Pendulum in cms.	Pendulum reading in degrees	Thickness of Pendulum in cms.	Pendulum reading in degrees.
0.405	55.0	0.052	34.8
1.002	22.5	0.090	43.2
1.099	38.6	0.202	51.1
1.302	53.3	0.298	56.1
0.801	51.2	0.950	42.3
0.896	49.2	1.051	29.2
0.597	56.9		

Table XXII.

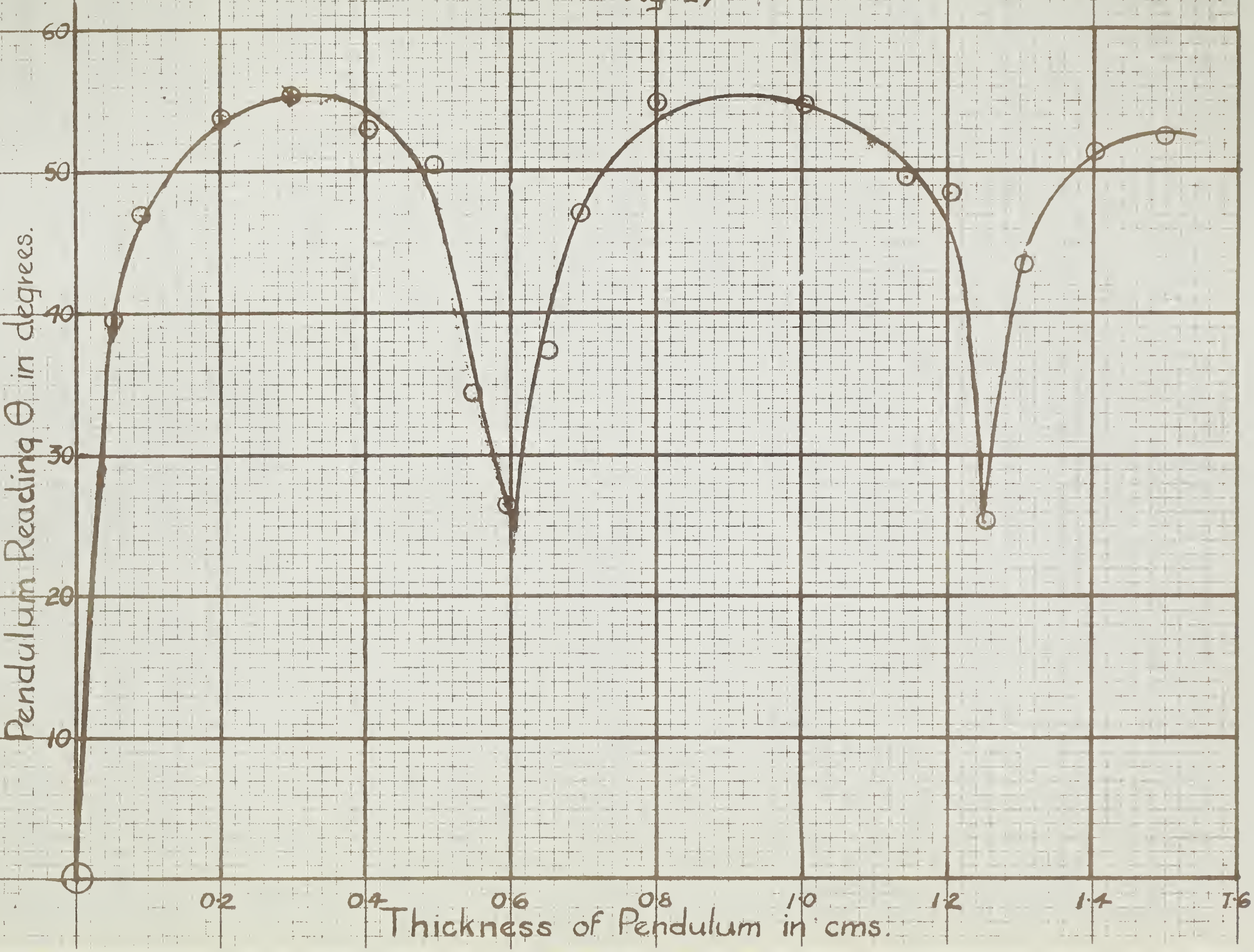
Frequency in cycles/sec.	Reading of Vane giving maximum reading in degrees.	Reading of vane near in thickness	Ratio of readings.
314500	38.0	18.8	0.495
315800	38.0	20.6	0.542
313200	35.1	15.3	0.436
311000	44.0	18.5	0.421
308900	50.0	21.0	0.420
309900	46.8	19.4	0.415
307900	56.3	24.5	0.435
306500	54.0	23.5	0.435

Table XXIII.

Maximum pendulum reading, 56.9° corresponds to the maximum value of R, 0.977 in the theoretical curve. λ_1 = 2.02 cms.

Thickness of Pendulum, L, in cms.	L/λ_1	Pendulum reading, θ, in degrees	R $(= \theta \times \frac{0.977}{56.9})$
0.405	0.200	55.0	0.946
1.002	0.497	22.5	0.387
1.099	0.544	38.6	0.664
1.302	0.645	53.3	0.917
0.801	0.396	51.2	0.880
0.896	0.444	49.2	0.846
0.597	0.296	56.9	0.977
0.052	0.0257	34.8	0.598
0.090	0.0446	43.2	0.742
0.202	0.100	51.1	0.879
0.298	0.148	56.1	0.965
0.950	0.470	42.3	0.727
1.051	0.521	29.2	0.502

Fig: 17


60
50
40
30
20
10
Pendulum Reading Θ in degrees.
0·2
0·4
0·6
0·8
1·0
1·2
1·4
1·6
Thickness of Pendulum in cms.

The values of the velocity were computed as in the case of lead. At a frequency of 531,200 cycles per sec., the velocity was 6.48×10^5 cms./sec., and at a frequency of 307,500 cycles/sec. the velocity was 6.21×10^5 cms./sec. A quoted velocity of low-frequency sound in duralumin, as determined by static methods, is 5.13×10^5 cms./sec.

In order to make certain that the velocities at lower frequencies were near those quoted, Young's Modulus and the density of a sample of the same material as used in the present experiments were determined. The modulus was taken by a tensile test in a Burkton testing machine+. The results were:

Young's Modulus, E, = $7.40 \times 10''$ dynes/cm.^{2};

Density, ρ_1 = 2.79 grams/cm.^{3}

Hence the velocity of sound in the material should be,

$$v = \sqrt{\frac{E}{\rho}} = \frac{7.40 \times 10''}{2.79} = 5.16 \times 10^5 \text{ cms./sec.}$$

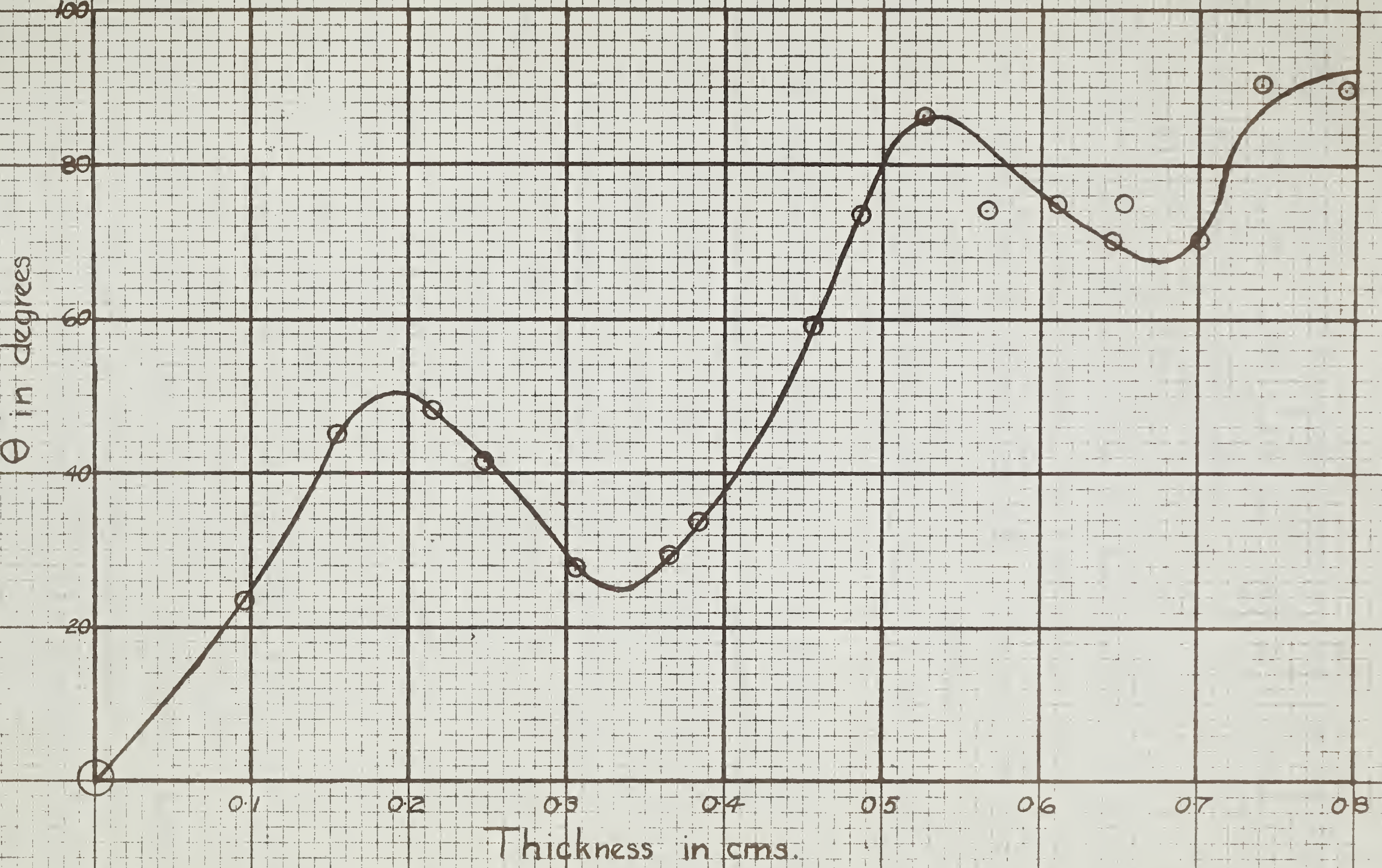
This is in good agreement with the quoted value of the velocity. The above formula is, however, applicable only to bodies with lateral dimensions small in comparison with a wave length, whereas the bodies in these experiments are of dimensions comparable with a wave length.

Paraffin.

A great deal of trouble was experienced with the pendulums of solid paraffin, which indeed proved a most unsatisfactory material with which to deal. However the work brought out some very interesting points. A series of paraffin pendulums would be used and the readings plotted against thickness. If this series were repeated some time later a totally different curve would often result. It was found, however, that consistent results could be obtained if the work were done under very stable conditions. Temperature especially affected the readings.

+ Indebted to Profs. Morrison and Cameron for this test.

Fig:20.


100
80
60
40
20
θ in degrees
0·1
0·2
0·3
0·4
0·5
0·6
0·7
0·8
Thickness in cms.

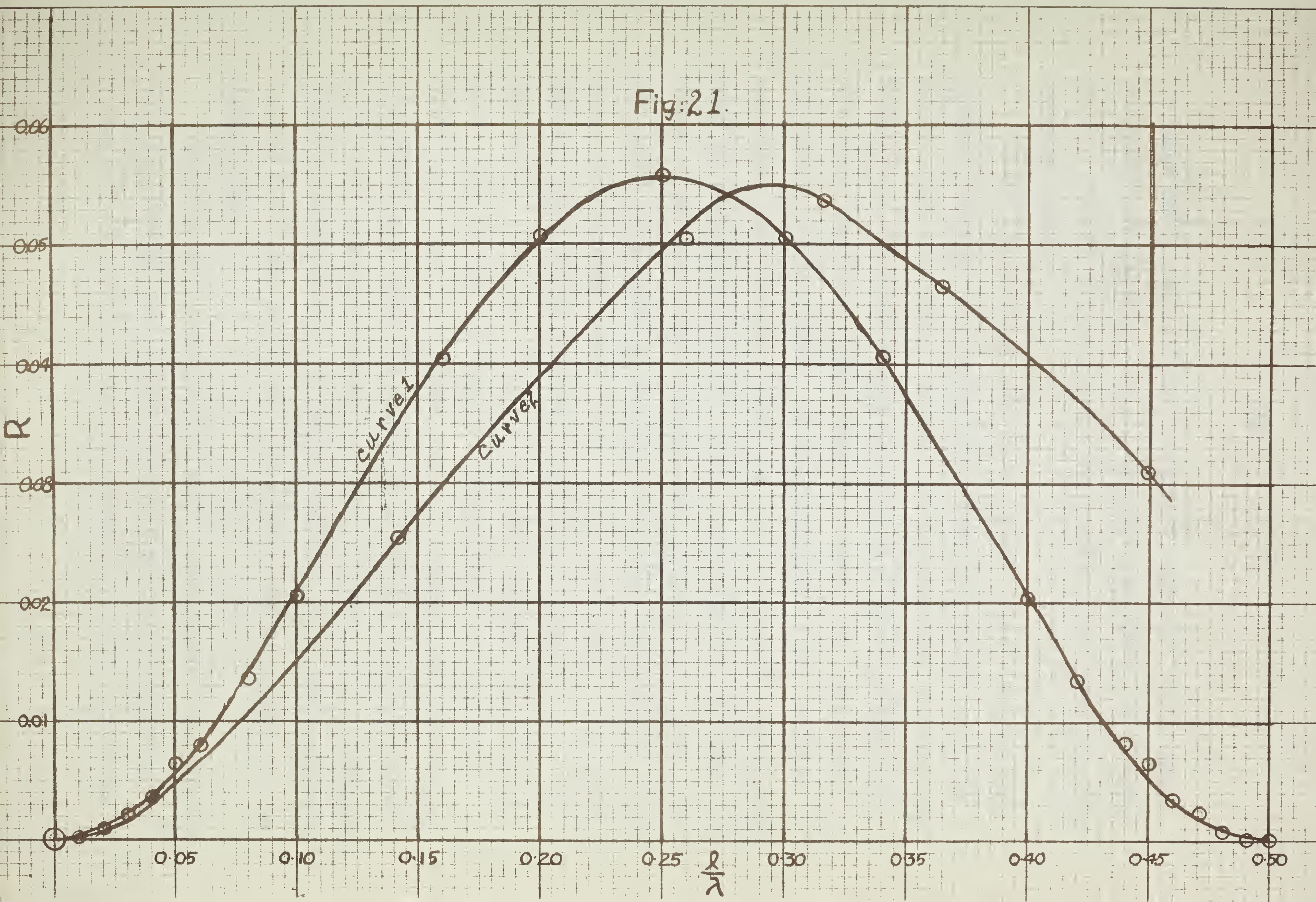
The elasticity of paraffin changes rapidly with the temperature at 12°C or 15°C, and consequently the velocity in sound changes rapidly as the temperature changes.

Paraffin is a good heat insulator and a pendulum of considerable thickness taken from air at say 20°C and placed in the water at about 12°C would not take up the temperature of the water throughout for some time, and, as far as its acoustic properties were concerned, would not be homogeneous. A thin pendulum would of course more quickly take up the prevailing temperature.

By keeping all of the pendulums in the water of the tank, and therefore at its temperature, and by taking a series of experiments very rapidly, before the temperature of the water could change appreciably, consistent results could be obtained.

A series of paraffin pendulum readings was taken with transmitter No. 2 operating at a frequency of 307500 cycles per second, and on checking were found to be consistent. Fig. 20 plotted from Table XXIV shows the pendulum readings plotted against thickness. The half wave length is about 0.34 cms. at a frequency of 307200 cycles per sec. Hence the velocity of the sound was $2 \times 0.4 \times 307200 = 2.09 \times 10^5$ cms./sec. The density of the paraffin was determined and found to be 0.901 grams/c.c. Using these values, the theoretical curve of R against L/λ_1, equation 4, was plotted in curve 1, fig. 21 from the computed values in Table XXV. The highest reading in the first half wave length, Table XXIV was taken as corresponding to the maximum value of R, and with 0.68 cms. as wavelength the readings for the first half wave length were reduced to ratios,

Fig:21.


R
0·06
0·05
0·04
0·03
0·02
0·01
0·05
0·10
0·15
0·20
0·25
0·30
0·35
0·40
0·45
0·50
$\frac{l}{\lambda}$
Curve 1
Curve 2

in Table XXVI, and plotted, in curve 2, fig. 21. The quoted value for the velocity, viz. 1.30 x 10^5 cms./sec., is in this case as well as in the case of the metals, much lower than the observed value.

It will be noticed that in the curve of pendulum reading against thickness the second loop rises much higher than the first. This was always found to be the case with paraffin and in one case three loops were obtained, ~~and~~ the second higher than the first, and the third higher than the second. An explanation can readily be conceived. As the thickness of the paraffin increased absorption of ultrasound in it increased thus cutting down the transmitted energy, which, in the case of paraffin, is always relatively large. The reflecting power of paraffin is low since the product of its density and the wave length of the sound in it is very nearly equal to the same product for water, and, as shown in the preceding theory, this would cause a low reflection. Hence as the thickness was increased the recoil pressure due to the transmitted energy became smaller on account of absorption. This, of course, caused a larger pendulum reading. Such conditions would not appear in metallic reflection, since most of the incident energy would be reflected, and the difference caused by absorption in the small quantity of energy transmitted would not be large enough to make appreciable difference in the observations.

It was thought advisable to see if the effects of harmonics could be detected and transmitter No. 3 was set oscillating at a frequency of 163200 cycles per second. The first and third

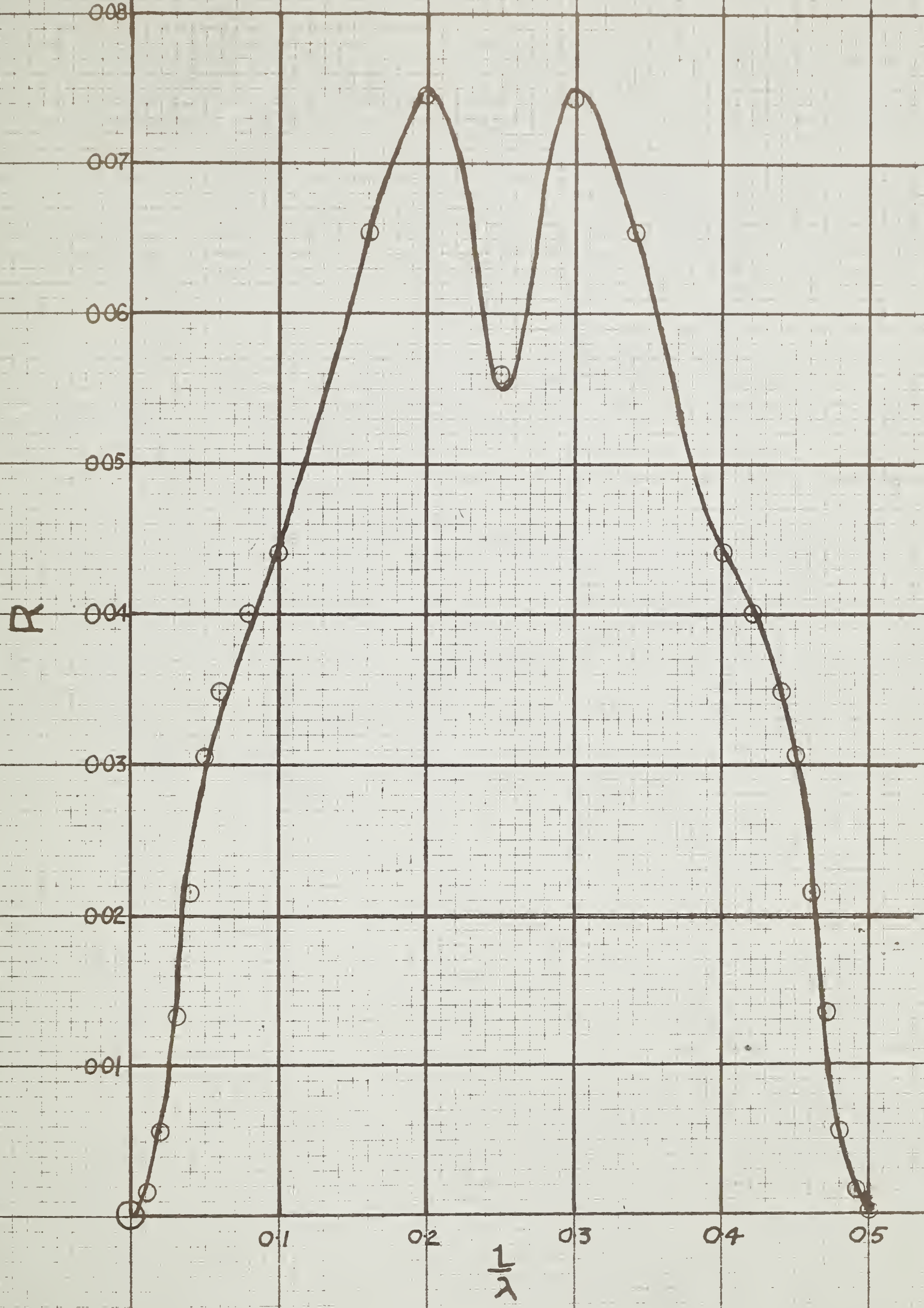


Fig: 22

Fig:

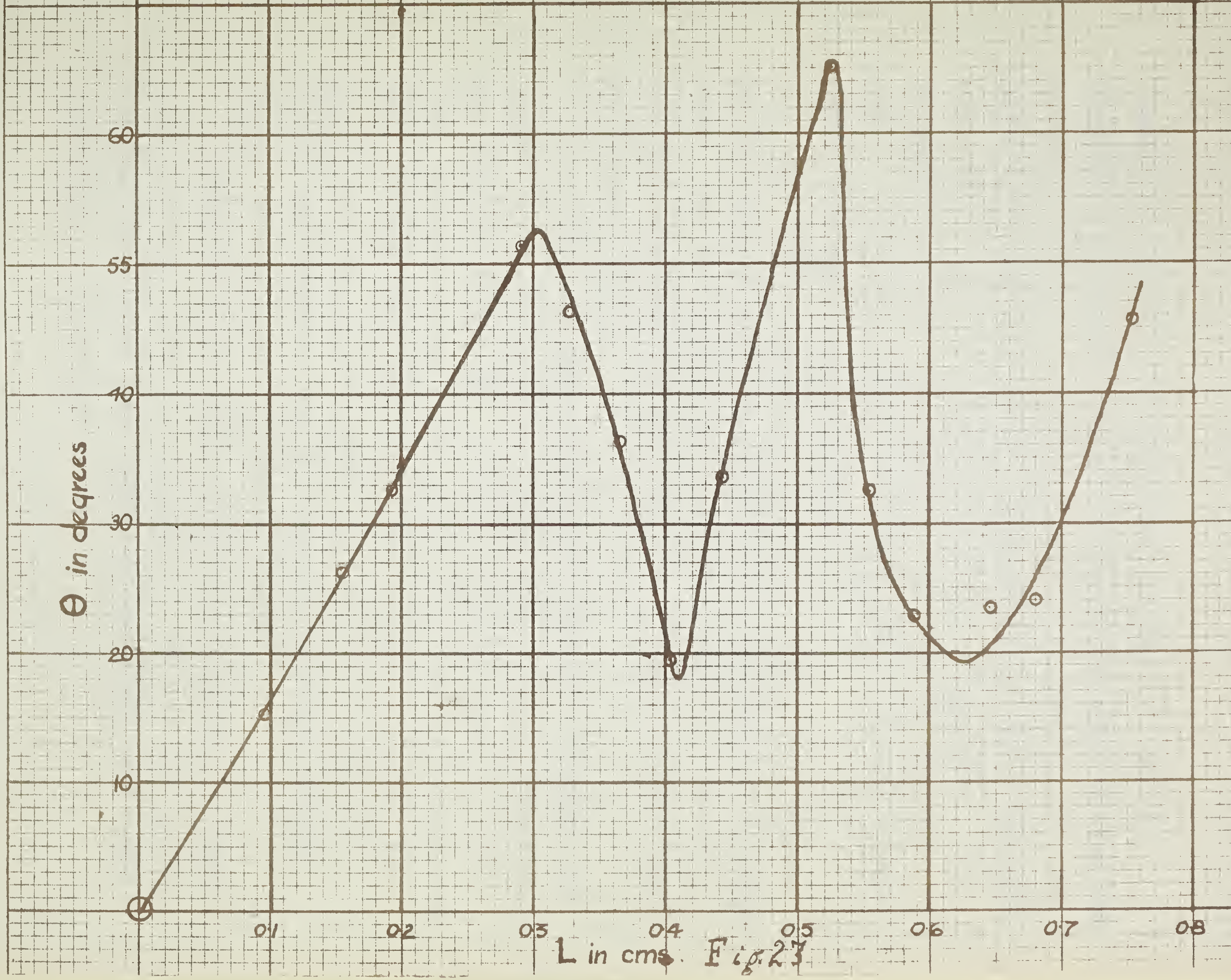


Fig. 27

electrical harmonics were found to be present but small in comparison with the fundamental. There was no means of detecting mechanical harmonics. It must be remembered, however, that the higher the frequency, the more concentrated the beam. The half angle of the beam, θ, is given by $\sin\theta = 1.22 \frac{\lambda}{d}$, where λ is the wave length and d the diameter of the face of the transmitter. Hence the effect of the harmonics is much more concentrated than that of the fundamental.

Fig. 22 shows R plotted against L/λ_1 using the theoretical relation, equation 14, plotted from Table XXVII for paraffin, where λ_1 is the fundamental wave length present and there are the first and third harmonics ~~present~~ each being 30% as intense as the fundamental.

A series of pendulum readings was taken at a frequency of 163200 cycles per second and plotted, from Table XXVIII, against the thickness in fig. 23. Although this curve does not ~~agree~~ with the curve in fig. 22 very well, it does show that harmonics have a marked effect upon reflection. It was, of course, impossible to determine the exact intensity of the harmonics present and whether or not any mechanical harmonics existed beyond the ones produced by the electrical.

According to the velocity determination above, the second dip in the curve, fig. 23 corresponds to one half wave length, and the other dip and extra peaks are undoubtedly due to some combination of harmonics.

Table XXIV.

Thickness of Pendulum in cms.	Pendulum Reading in degrees	Thickness of Pendulum in cms.	Pendulum Reading in degrees.
0.645	69.5		
0.610	75.0	0.365	29.2
0.652	75.0	0.154	45.1
0.566	74.0	0.215	48.0
0.700	69.6	0.248	41.6
0.739	90.4	0.456	59.2
0.525	86.2	0.487	73.5
0.305	27.7	0.793	89.4
0.383	33.3	0.097	23.5

Table XXV.

Deduced from relation $$R = \frac{\left(\frac{V\rho}{V_1\rho_1} - \frac{V_1\rho_1}{V\rho}\right)^2}{4\cot^2\frac{2\pi L}{\lambda_1} + \left(\frac{V\rho}{V_1\rho_1} + \frac{V_1\rho_1}{V\rho}\right)^2}$$

where $V = 1.48 \times 10^5$ cm./sec.,

$V_1 = 2.09 \times 10^5$ cms./sec.

$\rho = 1$ gram/cm.^{3}

$\rho_1 = 0.901$ grams/cm.^{3}

L/λ_1	$4\cot^2 2\pi L/\lambda_1$	R
0	∞	0.00000
0.01	1011	0.00023
0.02	251	0.00093
0.03	110	0.00207
0.04	60.7	0.00364
0.05	33.9	0.00620
0.06	25.6	0.00793
0.08	13.2	0.01358
0.10	7.26	0.02051
0.16	1.61	0.04050
0.20	0.422	0.0508
0.25	0	0.0559
0.30	0.422	0.0508
0.34	1.61	0.04050
0.40	7.62	0.02051
0.47	110	0.00207
0.475	159	0.00145
0.48	251	0.00093
0.49	1011	0.00023
0.50	∞	0.00000

Table XXVI.

Maximum deflection in first loop = 50^{0} and corresponds to $R = 0.0559$. $\lambda_1 = 0.680$ cms.

Thickness of pendulum, L, in cms.	Pendulum reading, θ, in degrees	$R \left(= \frac{0.0559}{50}\,\theta\right)$	L/λ_1
0.305	27.7	0.0310	0.499
0.365	29.2	0.0326	0.537
0.154	45.1	0.0504	0.226
0.215	48.0	0.0537	0.316
0.248	41.6	0.0465	0.365
0.097	23.5	0.0252	0.142

Table XXVII.

R_0 = ratio for fundamental, R_1 and R_3 = ratios for first and third harmonics respectively, each being $\frac{1}{3}$ as intense as the fundamental. λ_1 = the wave length of the fundamental.

L/λ_1	R_0	R_1	R_3	$R = R_0 + R_1 + R_3$
0.00	0.00000	0.00000	0.00000	0.00000
0.01	0.00023	0.00031	0.00121	0.00175
0.02	0.00093	0.00121	0.00453	0.00667
0.03	0.00207	0.00264	0.00860	0.01331
0.04	0.00364	0.00453	0.01350	0.02167
0.05	0.00670	0.00684	0.01693	0.03047
0.06	0.00793	0.00860	0.01833	0.03486
0.08	0.01358	0.01350	0.01300	0.04008
0.10	0.02051	0.01693	0.00684	0.04428
0.16	0.04050	0.01500	0.01000	0.08660
0.20	0.05080	0.00684	0.01693	0.07457
0.25	0.05590	0.00000	0.00000	0.05590

Table XXVIII.

Thickness of Pendulum, L, in cms.	Pendulum Reading θ, in degrees.	Thickness of Pendulum, L, in cms.	Pendulum Reading, θ, in degrees.
0.327	46.3	0.555	32.4
0.645	23.3	0.402	19.7
0.680	24.0	0.192	32.7
		0.299	51.4
0.095	15.2	0.154	26.4
0.524	65.0	0.365	36.2
0.443	33.6	0.752	45.6
0.589	22.7		

Below is given, in Table XXIX, a list of the velocities of sound in the different substances used together with the frequency at which they were determined and the values quoted in standard tables for the velocities.

Table XXIX.

Substance	Velocity of Sound Determined experimentally in cms./sec.	Velocity of Sound quoted in cms./sec.	Frequency at which Velocity was determined in cycles/sec.
Lead	2.23×10^5	1.227×10^5	531200
Lead	2.21×10^5	1.227×10^5	307500
Duralumin	6.48×10^5	5.13×10^5	531200
Duralumin	6.21×10^5	5.13×10^5	307500
Paraffin	2.09×10^5	1.304×10^5	307500

It will be noticed that the determined velocities are appreciably higher in all of these cases than the velocities quoted from static or low frequency methods. Also, although not marked in the case of lead, the velocity is higher the higher the frequency. The difference between the velocities at different frequencies is probably due to lateral effects of the pendulum vanes, which can be regarded as short rods the diameters of which were of the same order as the wave length. There is opened up here a problem which will need much further investigation.

Conclusions.

From the results obtained by the use of lead, duralumin and paraffin torsion pendulums of various thicknesses in the ultrasonic beam, it can be concluded:

(1) The theory outlined in this paper, although developed for a reflector of finite thickness and infinite extension, does fairly well indicate what actually takes place in reflection from and transmission through a reflector of finite dimensions. It is to be remarked that the agreement is so good in these experiments since here the linear dimensions of the reflectors were of the order of a wave length. Undoubtedly the reflection is a maximum and transmission a minimum for thicknesses equal to an odd number of quarter wave lengths, and reflection a minimum and transmission a maximum for thicknesses equal to an integral number of half wave lengths.

(2) The theory outlined contains no terms for absorption of the sound energy in the material of the partition. It has been shown that, in the case of paraffin, which is relatively a poor reflector of the ultrasonic energy, the effect of absorption can be demonstrated. As has been pointed out, the effect does not appear in the cases of the metals since their reflecting power is high and in consequence transmission small, and therefore any changes in the transmission due to absorption are not appreciable, whereas in the case of paraffin a proportionally large change would produce a marked change in the pendulum readings.

(3) The values of the velocity of sound in the different materials used as determined by the method outlined here are greater than values determined by static and low frequency methods. As explained above this is probably due to end effects in the pendulum vanes. It would be interesting to determine the velocities in large bodies of these materials at the same frequencies.

(4) In the three materials used the reflection ratio, R, at a thickness of one fourth wave length is higher the greater the difference between the products of density and wave length for the material and for the propagating medium. This result is indicated by the theory.

The author would like to acknowledge here the untiring assistance and cooperation of Dr. R. W. Boyle during the period of this research.

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